70: Discrete Math and Probability Theory

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Programming + Microprocessors $\equiv$ Superpower!

What are your super powerful programs/processors doing?
Logic and Proofs!
Induction $\equiv$ Recursion.

What can computers do?
Work with discrete objects.
Discrete Math $\implies$ immense application.

Computers learn and interact with the world?
E.g. machine learning, data analysis, robotics, ...
Probability!

The truth: My hopes and dreams.

You learn to think more clearly and more powerfully.

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You learn to think more clearly and more powerfully.

..And to deal precisely clearly with uncertainty itself.

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And to tell the truth.

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Truth??

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Truth?? Is there truth?

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Truth?? Is there truth? Evidence to decisions.

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Truth?? Is there truth? Evidence to decisions. What are values?

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Mathematical Reasoning is as close to truth as there is.

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It has a certain context.

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And it is (maybe) good to understand at least one context where it is solid.

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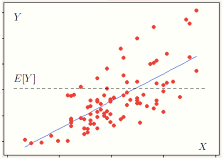
And the context vast consequences.

Probability Unit

- How can we predict unknown future events (e.g., gambling profit, next week rainfall, traffic congestion, …)?
  - Constructive Models: Model the overall system (including the sources of uncertainty).
    - For modeling uncertainty, we'll develop probabilistic models and techniques for analyzing them.
  - Deductive Models: Extract the "trend" from the previous outcomes (e.g., linear regression).



Learning.

Veritassium on Khan

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Confusion is the sweat of learning.

Learning.

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Confusion is the sweat of learning.

Confusion is the sweat of discovery.

Metacogition.

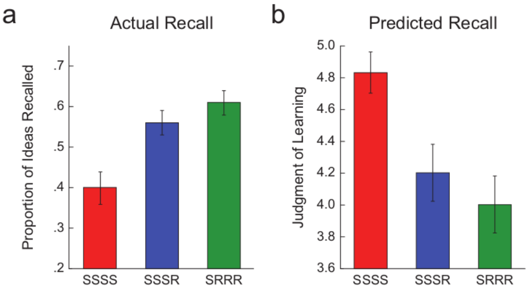

Fig. 1. Final recall (a) after repeatedly studying a text in four study periods (SSSS condition), reading a text in three study periods and then recalling it in one retrieval period (SSSR condition), or reading a text in one study period and then repeatedly recalling it in three retrieval periods (SRRR condition). Judgments of learning (b) were made on a 7-point scale, where 7 indicated that students believed they would remember material very well. The data presented in these graphs are adapted from Experiment 2 of Roediger and Karpicke (2006b). The pattern of students' metacognitive judgments of learning (predicted recall) was exactly the opposite of the pattern of students' actual long-term retention.

Learning styles.

How to search google.

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"Learning styles"

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  “Learning styles”
  “Learning styles debunked.”

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CS70: Notes, lectures, discussions, vitamins, homeworks.

An effective student is...

Smart, rich,

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Smart, rich,

and beautiful.

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All memes. The last one is not a meme.
First one, learning is inherent.

An effective student is...

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First one, learning is inherent. You are all capable.

Second, background, background, etc.

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Confident, motivated,

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What I think.

Confident, motivated,

has integrity.

Known knowns..

There are the known knowns, known unknowns, and unknown unknowns.

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The last one is what always gets you.

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In order to get there.

Dogs don’t have rights cuz..

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They don’t know infinity.

First grade

$1, 2, 3, 4, \ldots, 120$

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Peano's axioms. There is always a successor.

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$+3$ means move to successor and another and another, or 3 times.

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Obeys triangle inequality: $f(i,j) + f(j,k) \geq f(i,k)$

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11 is one ten, and one one.

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3×5?

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3×5?

$\times$ means add 3 times.
$5 + 5 + 5$
10 is moving over 5 from 5
The next number one can use the one's place.

How to interact with staff..

My advice to TA's.

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What should you do?

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What should you do?
Where does your understanding get iffy?

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E.g., Avoid long explanations with nodding students. You must checkin meaningfully.

What should you do?
  Where does your understanding get iffy?
    Explain what you understand, then say what you don't.

Advice from (former) TA's

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I read the notes until I could reproduce the proofs myself.

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Head TA Richard:
“carefully review the homework solutions after they are released and understand them to the point of being able to replicate them without needing to reference them.”

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Please do not take it out on your TA's.

Admin

Course Webpage: **http://www.eecs70.org/**

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Explains policies, has office hours, homework, midterm dates, etc.

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One midterm, final.

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Questions

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Questions $\implies$ Ed:

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Questions $\implies$ Ed:

Logistics, etc.

Content Support: other students!

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Announcements, logistics, critical advice.

Wason's experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.

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Suppose we have four cards on a table:

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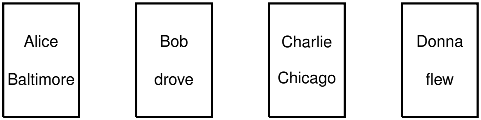
- Suppose you see that Alice went to Baltimore, Bob drove, Charlie went to Chicago, and Donna flew.



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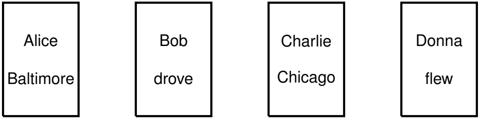

- Which cards must you flip to test the theory?

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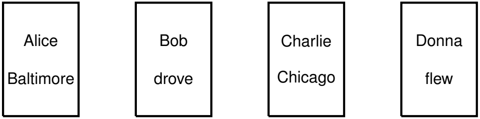


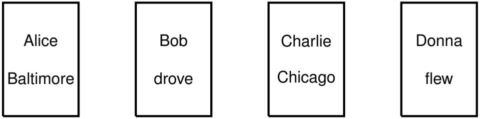
- Which cards must you flip to test the theory?

Answer: (A), (B), (C), (D).

Wason's experiment:1

Suppose we have four cards on a table:

- 1st about Alice, 2nd about Bob, 3rd Charlie, 4th Donna.
- Card contains person's destination on one side,
  and mode of travel.
- Consider the theory:
  "If a person travels to Chicago, they flies."
- Suppose you see that Alice went to Baltimore, Bob drove,
  Charlie went to Chicago, and Donna flew.



- Which cards must you flip to test the theory?

Answer: (A), (B), (C), (D). Later.

CS70: Lecture 1. Outline.

Today: Note 1.

CS70: Lecture 1. Outline.

Today: Note 1. Note 0 is background.

CS70: Lecture 1. Outline.

Today: Note 1.   Note 0 is background. Do read it.

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The language of proofs!

CS70: Lecture 1. Outline.

Today: Note 1.   Note 0 is background. Do read it.

The language of proofs!

1. Propositions.
2. Propositional Forms. (Formula.)
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan's Laws

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational
2+2 = 4
2+2 = 3
826th digit of pi is 4
Jesse Eisenberg is a good actor
Any even > 2 is sum of 2 primes
$4 + 5$
$x + x$
Alice travelled to Chicago

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I love you.	**Hmmm.**	Its complicated.

Again: "value" of a proposition is ... True or False

Propositional Forms.

Put propositions together to make another...

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Conjunction ("and"): $P \wedge Q$

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$\neg$ “$(2+2=4)$” – a proposition that is ...

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Put them together..

Propositions:

P_1 - Person 1 rides the bus.

Put them together..

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But we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 rides the bus or person 5 doesn't.

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Can person 3 ride the bus?

Put them together..

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Can person 3 and person 4 ride the bus together?

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This seems ...

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We can program!!!!

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We can program!!!!

We need a way to keep track!

Truth Tables for Propositional Forms.

"$P \wedge Q$" is True if
both P and Q are True .

P	Q	$P \wedge Q$
T	T	T
T	F	
F	T	
F	F	

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Check: $\wedge$ and $\vee$ are commutative.

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Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

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Check: $\wedge$ and $\vee$ are commutative.

One use for truth tables: Logical Equivalence of propositional forms!

Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$.

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DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q)$

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T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q) \quad \equiv \quad \neg P \vee \neg Q \qquad\qquad \neg(P \vee Q) \quad \equiv \quad \neg P \wedge \neg Q$

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Is $(T \wedge Q) \equiv Q$?

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
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P	Q	$P \vee Q$
T	T	T
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Is $(T \wedge Q) \equiv Q$? Yes? No?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
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P	Q	$P \vee Q$
T	T	T
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Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes!

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
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T	T	T
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Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$? T

Quick Questions

P	Q	$P \wedge Q$
T	T	T
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T	T	T
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Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$? T

What is $(F \vee Q)$?

Quick Questions

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Is $(T \wedge Q) \equiv Q$? Yes? No?

Yes! Look at rows in truth table for $P = T$.

What is $(F \wedge Q)$? F or False.

What is $(T \vee Q)$? T

What is $(F \vee Q)$? Q

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

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$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$,

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

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$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

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Distributive?

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P is False .

Distributive?

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Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

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P is False .

LHS: $F \wedge (Q \vee R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

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P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

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RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F)$

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:
P is True .
LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.
RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.
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LHS: $F \wedge (Q \vee R) \equiv F$.
RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:
P is True .
LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.
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P is False .
LHS: $F \wedge (Q \vee R) \equiv F$.
RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

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RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$,

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:
P is True .
LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.
RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.
P is False .
LHS: $F \wedge (Q \vee R) \equiv F$.
RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

Foil 2:

Distributive?

$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$?

Simplify: $(T \wedge Q) \equiv Q$, $(F \wedge Q) \equiv F$.

Cases:

P is True .

LHS: $T \wedge (Q \vee R) \equiv (Q \vee R)$.

RHS: $(T \wedge Q) \vee (T \wedge R) \equiv (Q \vee R)$.

P is False .

LHS: $F \wedge (Q \vee R) \equiv F$.

RHS: $(F \wedge Q) \vee (F \wedge R) \equiv (F \vee F) \equiv F$.

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$?

Simplify: $T \vee Q \equiv T$, $F \vee Q \equiv Q$. ...

Foil 1:

$(A \vee B) \wedge (C \vee D) \equiv (A \wedge C) \vee (A \wedge D) \vee (B \wedge C) \vee (B \wedge D)$?

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$(A \wedge B) \vee (C \wedge D) \equiv (A \vee C) \wedge (A \vee D) \wedge (B \vee C) \wedge (B \vee D)$?

Implication.

$P \implies Q$ interpreted as

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If P, then Q.

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True Statements: P, $P \implies Q$.

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Implication.

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Examples:

Statement: If you stand in the rain, then you'll get wet.

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Conclude: Q is true.

Examples:

Statement: If you stand in the rain, then you'll get wet.
P = "you stand in the rain"

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Statement: If you stand in the rain, then you'll get wet.
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Statement: "Stand in the rain"

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Statement: If you stand in the rain, then you'll get wet.
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Statement: "Stand in the rain"
Can conclude: "you'll get wet."

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Statement:
If a right triangle has sidelengths $a \leq b \leq c$, then $a^2 + b^2 = c^2$.

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Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

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The statement "$P \implies Q$"

only is False if P is True and Q is False .

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Implication and English.

$P \implies Q$

Poll.

- If P, then Q.

Implication and English.

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Implication and English.

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Poll.

- If P, then Q.
- Q if P.
  Just reversing the order.
- P only if Q.

Implication and English.

$P \implies Q$

Poll.

- If P, then Q.
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  Just reversing the order.
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  Remember if P is true then Q must be true.

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  since if Q is false P must have been false.

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- P is sufficient for Q.

Implication and English.

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  Example: Showing $n > 4$ is sufficient for showing $n > 3$.

Implication and English.

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  Example: Showing $n > 4$ is sufficient for showing $n > 3$.
- Q is necessary for P.
  For P to be true it is necessary that Q is true.
  Or if Q is false then we know that P is false.

Implication and English.

$P \implies Q$

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  This means that proving P allows you
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  Example: Showing $n > 4$ is sufficient for showing $n > 3$.
- Q is necessary for P.
  For P to be true it is necessary that Q is true.
  Or if Q is false then we know that P is false.
  Example: It is necessary that $n > 3$ for $n > 4$.

Truth Table: implication.

P	Q	$P \implies Q$
T	T	T
T	F	
F	T	
F	F	

Truth Table: implication.

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P	Q	$\neg P \vee Q$
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Truth Table: implication.

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$\neg P \vee Q \quad \equiv P \implies Q.$

These two propositional forms are logically equivalent!

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.

Contrapositive, Converse

- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
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- **Converse** of $P \implies Q$ is $Q \implies P$.

  If fish die the plant pollutes.

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  Logically equivalent! Notation: $\equiv$. Recall: $(X \implies Y) \equiv (\neg X \vee Y)$
  $P \implies Q \equiv \neg P \vee Q \equiv Q \vee \neg P \equiv \neg(\neg Q) \vee \neg P \equiv \neg Q \implies \neg P$.
- **Converse** of $P \implies Q$ is $Q \implies P$.

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- **Contrapositive** of $P \implies Q$ is $\neg Q \implies \neg P$.
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- **Definition:** If $P \implies Q$ and $Q \implies P$ is P if and only if Q or $P \iff Q$.
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Next: Statements about boolean valued functions!!

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Universe examples include..

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$Flew(B)$ = False . Do we care about $Chicago(B)$?
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Theory: "If a person travels to Chicago, he/she/they flies."

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Only have to turn over cards for Bob and Charlie.

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Later we may omit universe if clear from context.

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Theorem: $(\forall n \in \mathbb{N})\ n \geq 3 \implies \neg(\exists a, b, c \in \mathbb{N})\ (a^n + b^n = c^n)$

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1993: Wiles ...(based in part on Ribet's Theorem)

DeMorgan Restatement:
Theorem: $\neg(\exists n \in \mathbb{N})\ (\exists a, b, c \in \mathbb{N})\ (n \geq 3 \implies a^n + b^n = c^n)$

Summary.

Propositions are statements that are true or false.

Propositional forms use $\wedge, \vee, \neg$.

Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

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Next Time: proofs!