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"Period" divides $p-1$.

Today.

Today.

Polynomials.

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Polynomials.

Secret Sharing.

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Correcting for loss or even corruption.

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Share secret among n people.

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Two points make a line.

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The idea of the day.

Two points make a line.
Lots of lines go through one point.

Polynomials

A **polynomial**

$$P(x) = a_d x^d + a_{d-1} x^{d-1} \cdots + a_0.$$

is specified by **coefficients** $a_d, \ldots a_0$.

[1] A field is a set of elements with addition and multiplication operations, with inverses. $GF(p) = (\{0, \ldots, p-1\}, + \pmod{p}, * \pmod{p})$.

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Note: Often polynomial of degree d means polynomial of at most d.

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Polynomial: $P(x) = a_d x^4 + \cdots + a_0$

Line:$P(x) = a_1 x + a_0$

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Line:$P(x) = a_1 x + a_0 = mx + b$

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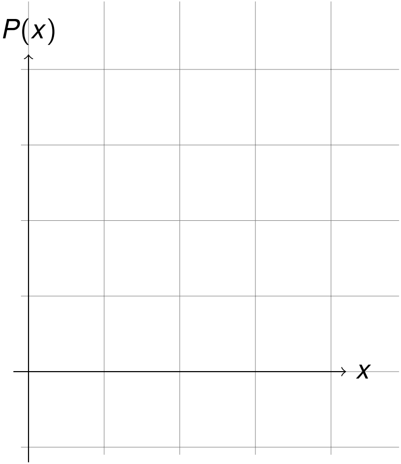

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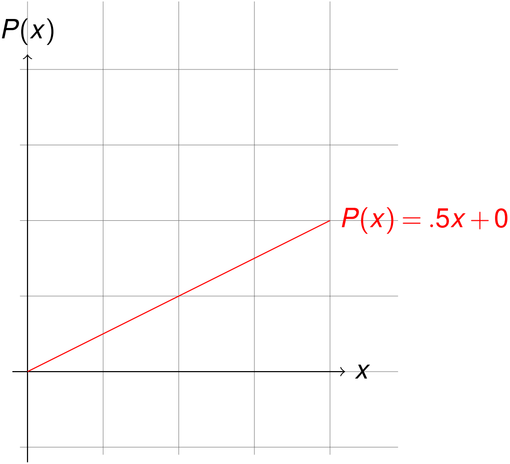

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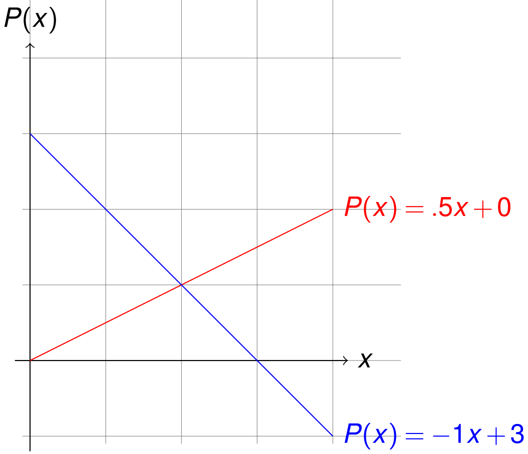

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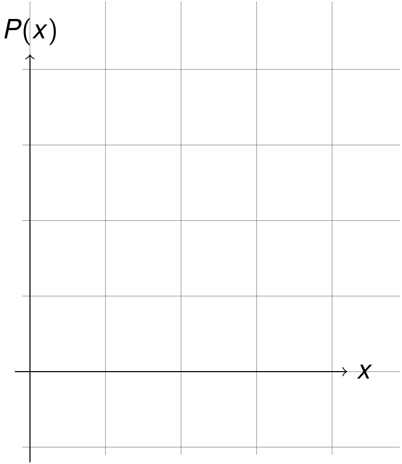

Parabola: $P(x) = a_2 x^2 + a_1 x + a_0$

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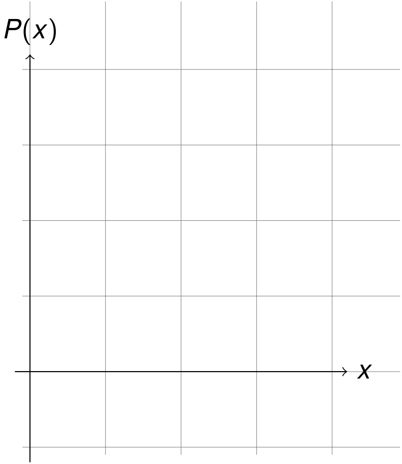

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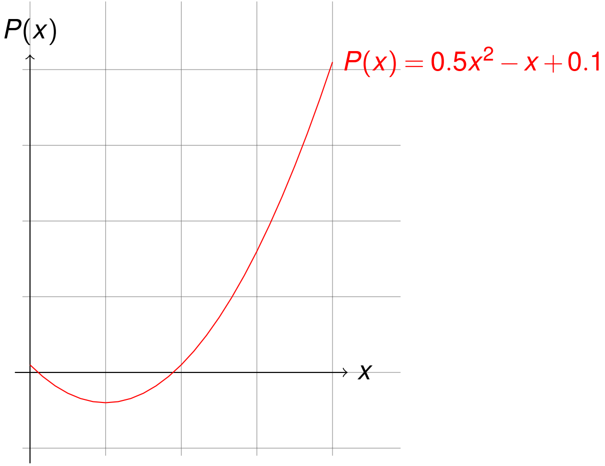

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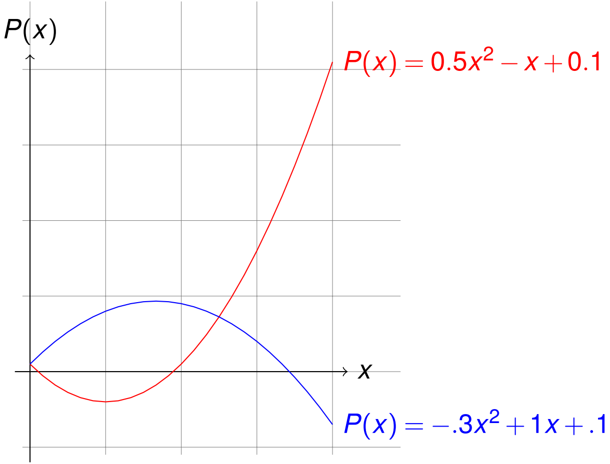

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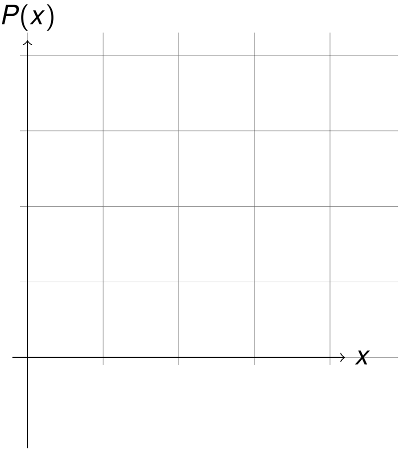

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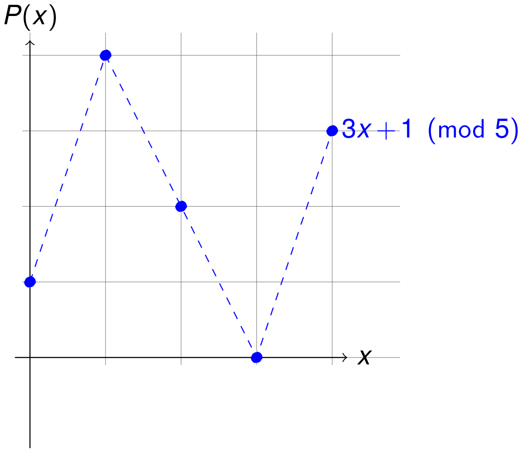

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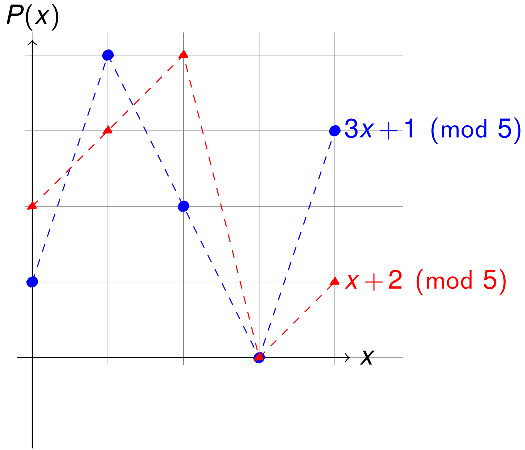

Finding an intersection.

$x + 2 \equiv 3x + 1 \pmod{5}$

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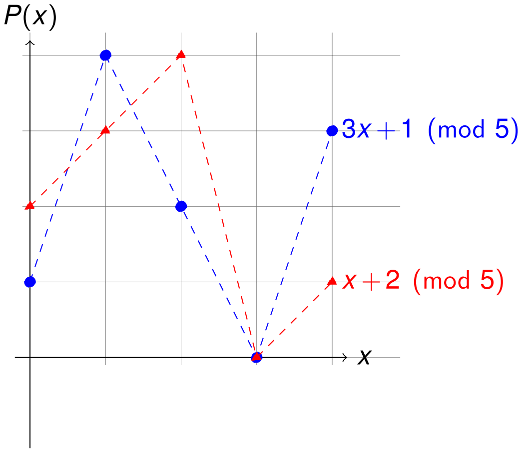

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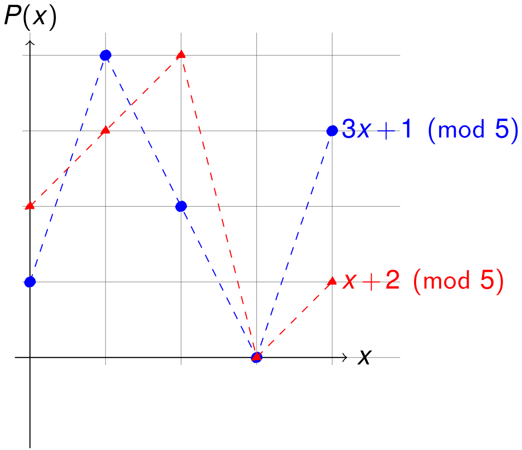

Finding an intersection.
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3 is multiplicative inverse of 2 modulo 5.
Good when modulus is prime!!

Two points make a line.

Fact: Exactly 1 degree $\leq d$ polynomial contains $d+1$ points. [2]

[2] Points with different x values.

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Modular Arithmetic Fact: Exactly 1 degree $\leq d$ polynomial with arithmetic modulo prime p contains $d+1$ pts.

---

[2] Points with different x values.

Poll.

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What facts below tell you this?

Say points are $(x_1, y_1), (x_2, y_2)$**.**

Poll.

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(C) The unknowns are m and b.
(D) If two equations have unique solution, done.

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All true.

In the Flow (Steph Curry) Poll.

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Flow poll. (All true. (B) is not a proof, it is restatement.)

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(A) $a_1 = m$
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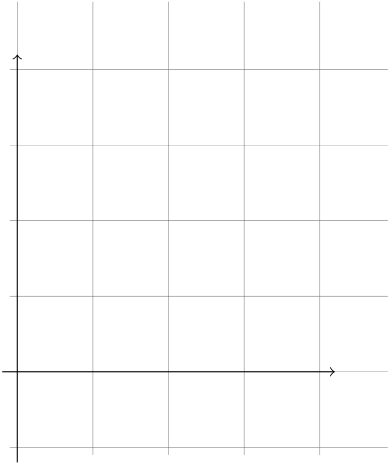
(A) and (D)

3 points determine a parabola.



Fact: Exactly 1 degree $\leq d$ polynomial contains $d+1$ points. [3]

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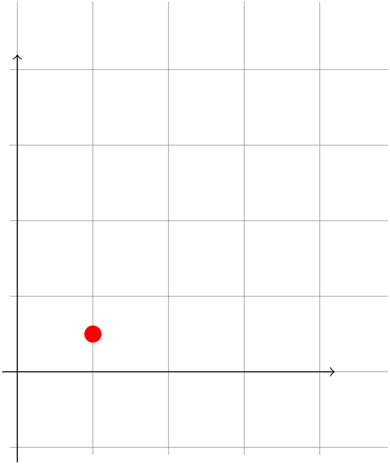

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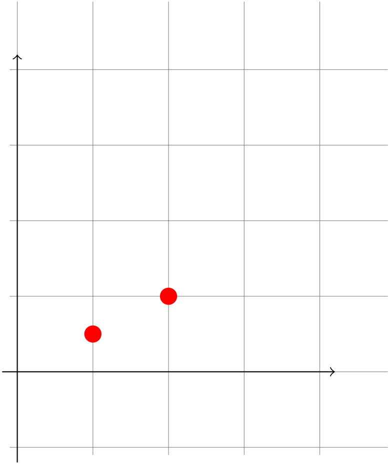

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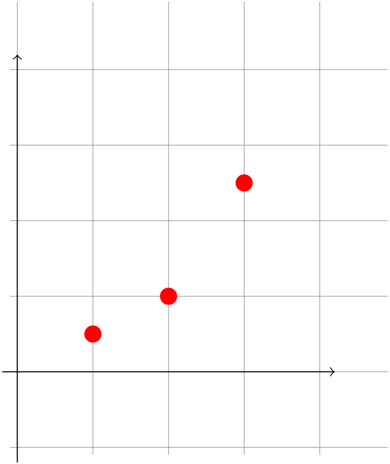

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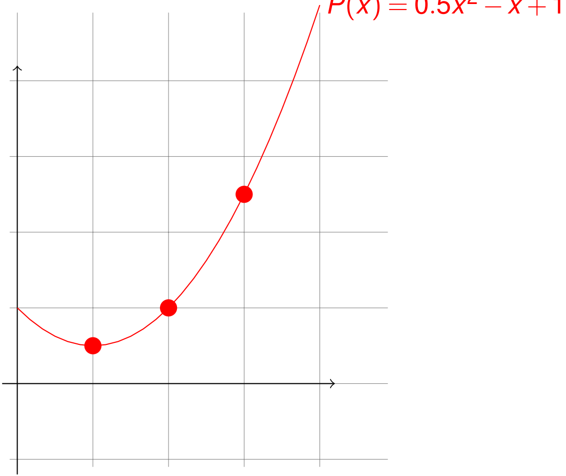

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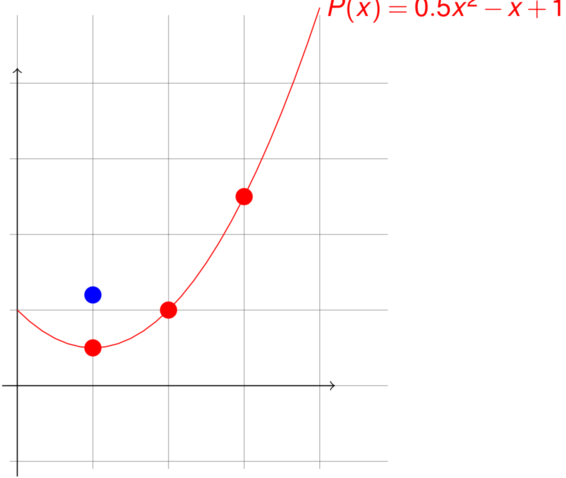

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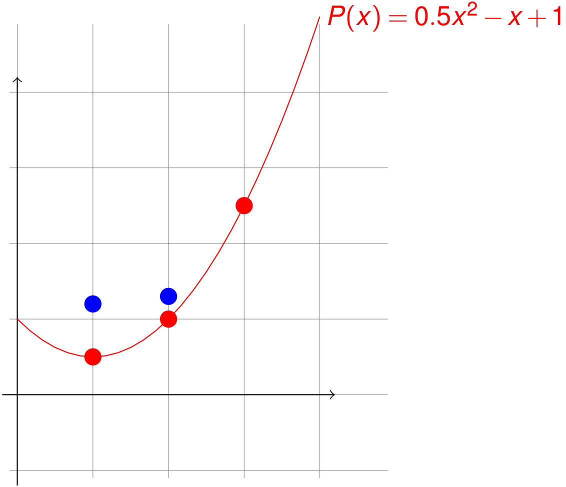

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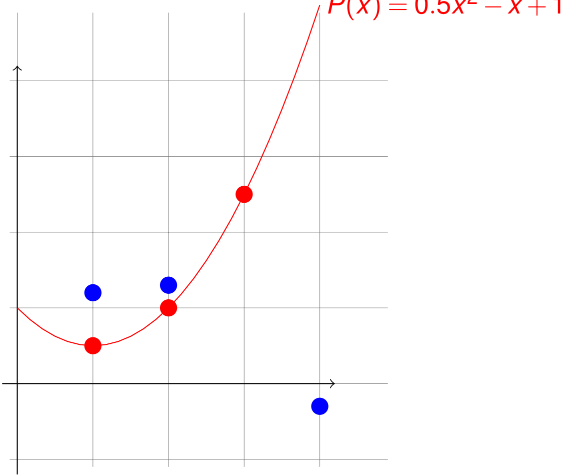

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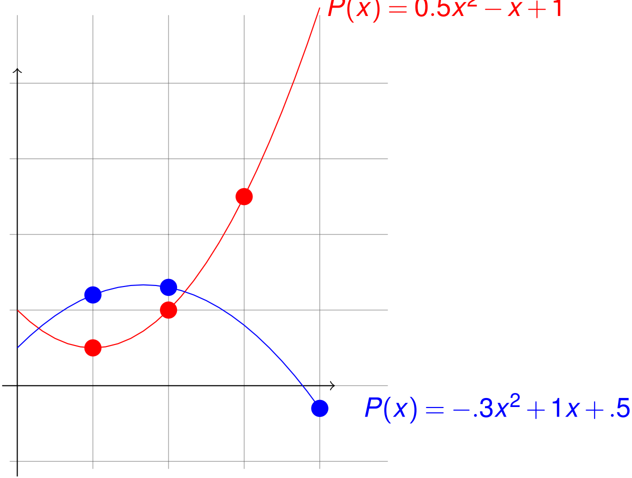

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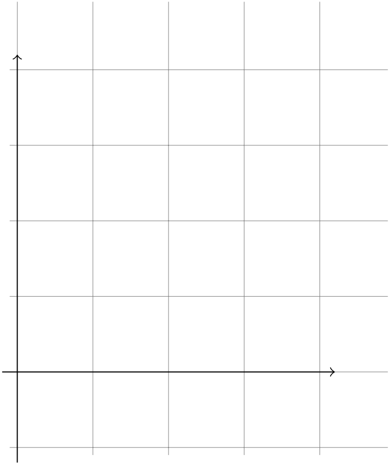

There is $P(x)$ contains blue points and *any* $(0, y)$!

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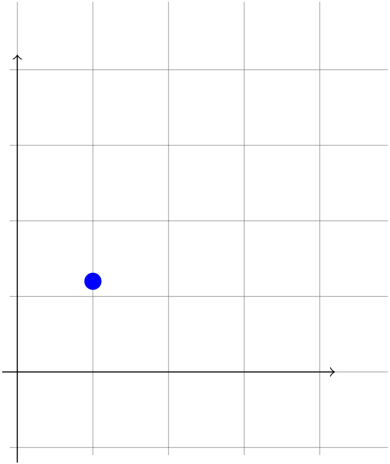

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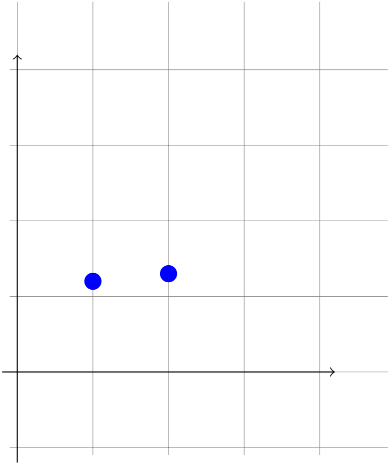

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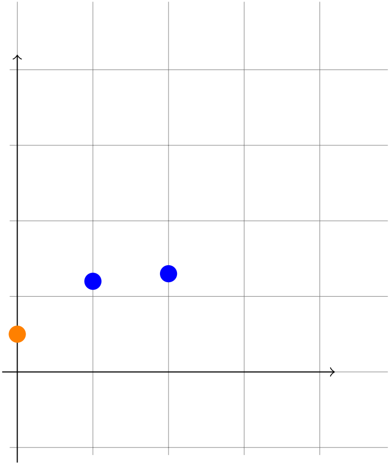

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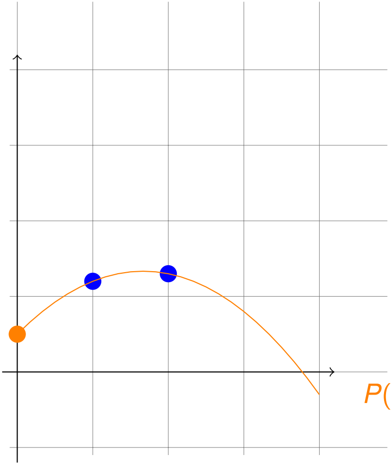

$P(x) = -.3x^2 + 1x + .5$

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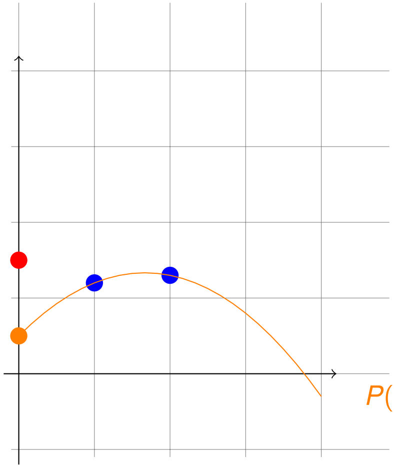

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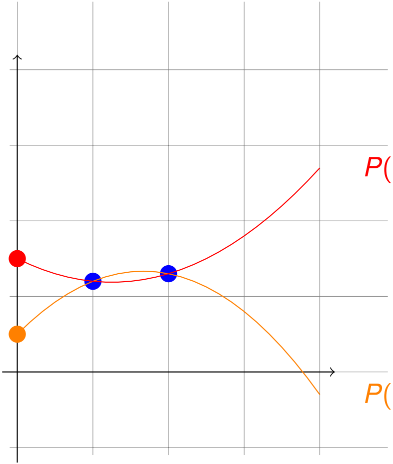

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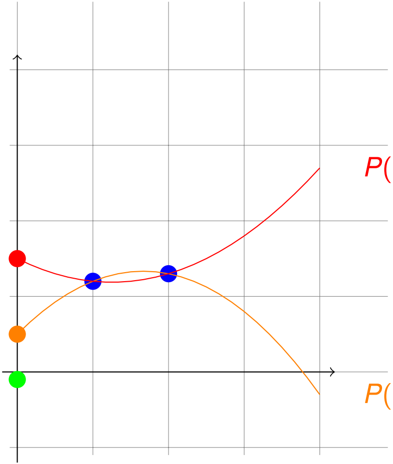

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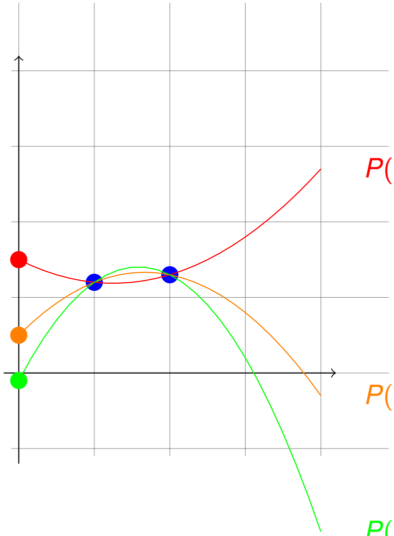

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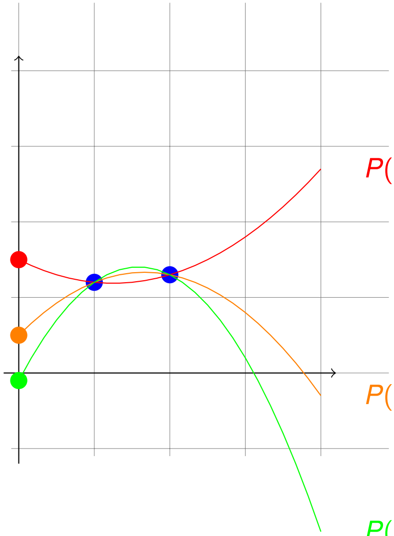

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3. Share i is point $(i, P(i) \mod p)$.

Roubustness: Any k shares gives secret.
Knowing k pts $\implies$ only one $P(x)$ $\implies$ evaluate $P(0)$.
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Modular Arithmetic Fact and Secrets

Modular Arithmetic Fact: Exactly 1 degree $\leq d$ polynomial with arithmetic modulo prime p contains $d+1$ pts.

Shamir's k out of n Scheme:
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So polynomial is $2x^2 + 1x + 4 \pmod 5$

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In general, divide $P(x)$ by $(x - a)$ gives $Q(x)$ and remainder r.

That is, $P(x) = (x - a)Q(x) + r$ where $Q(x)$ has degree $d - 1$.

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Intuitively, a field is a set with operations corresponding to addition, multiplication, and division.

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Modular Arithmetic Fact: Exactly one polynomial degree $\leq d$ over $GF(p)$, $P(x)$, that hits $d+1$ points.

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(Almost) the same as what is missing: one $P(i)$.

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Runtime: polynomial in k, n, and $\log p$.

1. Evaluate degree $k-1$ polynomial n times using $\log p$-bit numbers.
2. Reconstruct secret by solving system of k equations using $\log p$-bit arithmetic.

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Secret Sharing:
k points on degree $k-1$ polynomial is great!
Can hand out n points on polynomial as shares.