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Recall: powerset of the naturals is not countable.

Resolution of hypothesis?

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Can't use math!

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Uh oh....

Is it actually useful?

Write me a program checker!

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Check that the compiler works!

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How about.. Check that the compiler terminates on a certain input.

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How long do you wait?

Something about infinity here, maybe?

Halt does not exist.

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Proof: Yes! No! Yes! No! No! Yes! No! Yes! .. $\square$

Yes! No!...

What is he talking about?

Yes! No!...

What is he talking about?

(A) He is confused.

Yes! No!...

What is he talking about?

(A) He is confused.
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(B) and (D) maybe?

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(B) and (D) maybe? and maybe (A).

Professor does love Welch-Berlekamp though!

Halt and Turing.

Proof:

Halt and Turing.

Proof: Assume there is a program $HALT(\cdot,\cdot)$.

Halt and Turing.

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Turing(P)

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Contradiction. Program HALT does not exist!

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Questions?

Another view of proof: diagonalization.

Any program is a fixed length string.

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Fixed length strings are enumerable.

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Any program is a fixed length string.
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Program halts or not on any input, which is a string.

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Program halts or not on any input, which is a string.

	P_1	P_2	P_3	$\cdots$
P_1	H	H	L	$\cdots$
P_2	L	L	H	$\cdots$
P_3	L	H	H	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

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Turing - is not Halt.

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Turing is not on list.

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Halt - diagonal.
Turing - is not Halt.
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Turing is not on list. Turing is not a program.

Another view of proof: diagonalization.

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and is different from every P_i on the diagonal.
Turing is not on list. Turing is not a program.
Turing can be constructed from Halt.

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Fixed length strings are enumerable.
Program halts or not on any input, which is a string.

	P_1	P_2	P_3	$\cdots$
P_1	H	H	L	$\cdots$
P_2	L	L	H	$\cdots$
P_3	L	H	H	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

Halt - diagonal.
Turing - is not Halt.
and is different from every P_i on the diagonal.
Turing is not on list. Turing is not a program.
Turing can be constructed from Halt.
Halt does not exist!

Another view of proof: diagonalization.

Any program is a fixed length string.
Fixed length strings are enumerable.
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Programs?

What are programs?

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(A) Instructions.
(B) Text.
(C) Binary String.
(D) They run on computers.

Programs?

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(A) Instructions.
(B) Text.
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All are correct.

Proof play by play.

Assumed HALT(P, I) existed.

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What is P?

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$\Longrightarrow$ HALT is not a program.

Questions?

We are so smart!

Wow, that was easy!

We are so smart!

Wow, that was easy!

We should be famous!

No computers for Turing!

In Turing's time.

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In Turing’s time.

No computers.

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Adding machines.

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Concept of program as data wasn't really there.

Turing machine.

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A Turing machine.
– an (infinite) tape with characters

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Turing: AI, self modifying code, learning...

Turing and computing.

Just a mathematician?

Turing and computing.

Just a mathematician?

“Wrote” a chess program.

Turing and computing.

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“Wrote” a chess program.

Simulated the program by hand to play chess.

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The polish machine...the *bomba.*

Computing on top of computing...

Computer, assembly code, programming language, browser, html, javascript..

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Computer, assembly code, programming language, browser, html, javascript..

We can't get enough of building more Turing machines.

Reductions from HALT.

Does a program, P, print “Hello World” on input x?

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Does P, x halt?
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  Make P' from P

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 Does P, x halt?
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    How: rewrite text of P to form P'.

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Thus, HALT is HelloWorld(P')

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Thus, HALT is HelloWorld(P’) and $P' = \textit{Reduce}(P)$.

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But HALT does not exist $\implies$ "HELLO, WORLD?" does not exist.

Reduce HALT to HELLOWORLD?

Does P Halt on x?

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Rewrite to P'.

Reduce HALT to HELLOWORLD?

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How?

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How? What is P?

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How? What is P? Text!!!!!!

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Rewrite:

Reduce HALT to HELLOWORLD?

Does P Halt on x?

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Rewrite:
  Remove all print statements.

Reduce HALT to HELLOWORLD?

Does P Halt on x?

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Rewrite:
  Remove all print statements.
  Find exit points and add statement: **Print** “Hello World.”

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  If: If P halts, P' prints “Hello World.”

Subroutine version of reduction.

Input to HALT: P.

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Program Reduce takes P and writes this program.

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```
def P'(x):
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```
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   Restore stdout.
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$P'(x)$ prints "Hello World" if and only if $P(x)$ halts.

Undecidable problems.

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Computing as a lense: Science, Quantum Computing, DNA, ...

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Tragic ending...

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- British Government apologized (2009) and pardoned (2013).

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Summary: decidability.

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Computation is a lens for other action in the world.

Kolmogorov Complexity, Google, and CS70

Of strings, s.

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Minimum sized program that prints string s.

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for $i = 1$ to n: print '1'.

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What is the minimum I need to know (remember) to know stuff.

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Radius of the earth?

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perimeter of a circle/diameter.

Kolmogorov Complexity, Google, and CS70

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Calculus: what is minimum you need to know?
Depends on your skills! Conceptualization.
Reason and understand an argument and you can generate a lot.

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What is the first half of calculus about?

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The slope of a tangent line to a function at a point.

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Idea: use rise in function value!

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Any concept:

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Any concept:
A quick argument from basic concept of slope of a tangent line.

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Perhaps.

Derivative of sine?

$\sin(x)$.

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What is x? An angle in radians.

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Let's call it θ and do derivative of $\sin\theta$.

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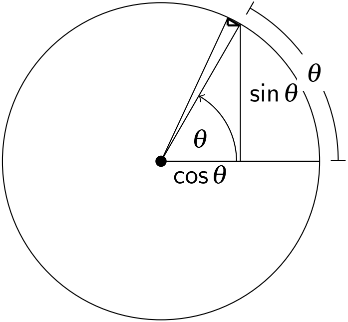

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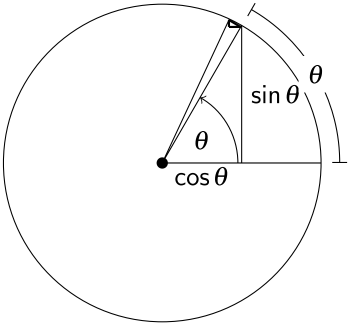

Rise.

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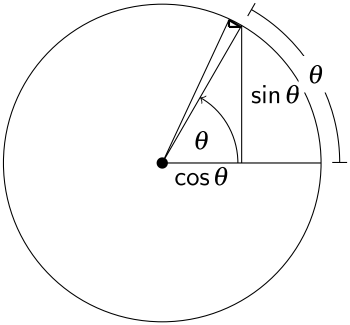

Rise.
Similar triangle!!!

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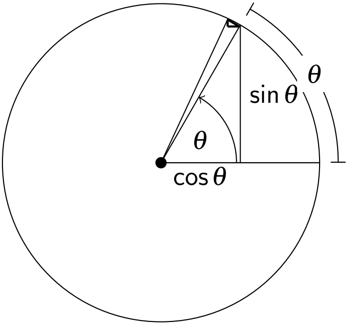

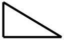

Rise.
Similar triangle!!!
Rise proportional to cosine!

Fundamental Theorem of Calculus.

Conceptual: Height times Width = Area.

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Useful?

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Useful?
  Speed times Time is Distance.

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Conceptual: Area is proportional to height.

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If you change width, change in area is proportional to height.

Fundamental Theorem of Calculus.

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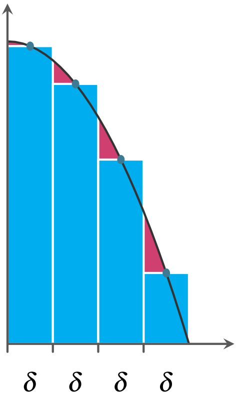
Derivative (rate of change) of Area (Integral) under curve, is height of curve.

Calculus



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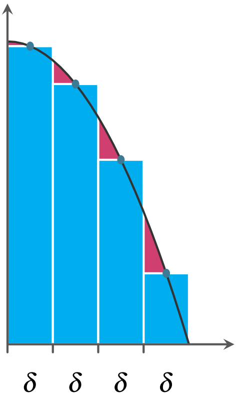
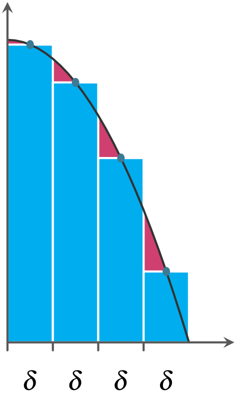


Riemann Sum/Integral: $\int_a^b f(x)dx = lim_{\delta \to 0} \sum_i \delta f(a_i)$

Calculus



Riemann Sum/Integral: $\int_a^b f(x)dx = lim_{\delta \to 0} \sum_i \delta f(a_i)$
"Area is defined as rectangles and add up some thin ones."

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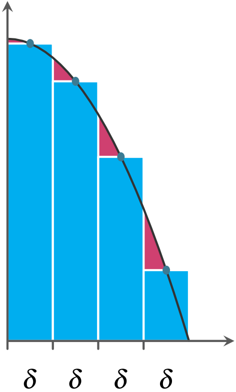

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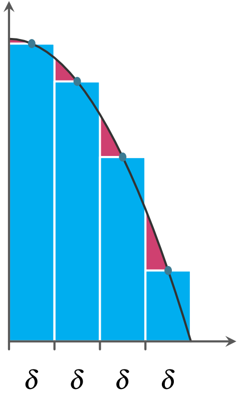

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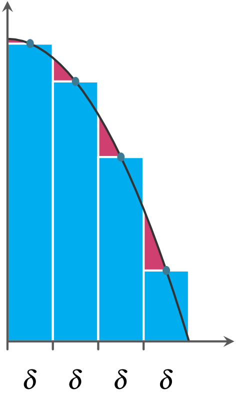

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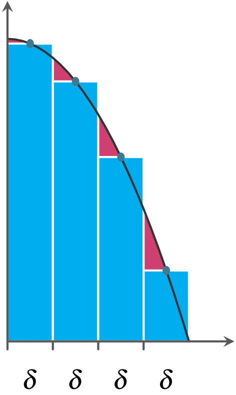

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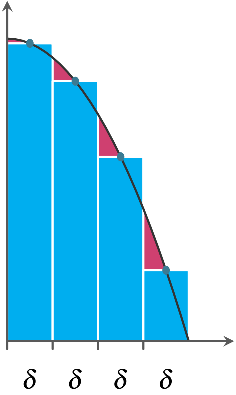

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"Area ($F(\cdot)$) under $f(x)$ grows at x, $F'(x)$, by $f(x)$"
Thus $F'(x) = f(x)$.

Arguments, reasoning.

What you know: slope, limit.

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Knowing how to program

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Knowing how to program plus some syntax (google) gives the ability to program.

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Discrete Math: basics are counting, how many, when are two sets the same size?

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Probability: division.

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CS 70 : ideas.

Induction

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- Euler's formula: tree has $v - 1$ edges and 1 face plus
  - each extra edge makes additional face.
  - $v - 1 + (f - 1) = e$

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Number theory.

A divisor of x and y divides $x - y$.

The remainder is always smaller than the divisor.

$\Longrightarrow$ Euclid's GCD algorithm.

Multiplicative Inverse.

Fermat's theorem: function with inverse is a bijection.

Multiplication is commutative.

Gives RSA.

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The remainder is always smaller than the divisor.
$\implies$ Euclid's GCD algorithm.
Multiplicative Inverse.
Fermat's theorem: function with inverse is a bijection.
Multiplication is commutative.
Gives RSA.

Error Correction.
(Any) Two points determine a line.
(well, and d points determine a degree $d+1$-polynomials.
Cuz, factoring.
Find line by linear equations.
If an equation is wrong, then multiply them by zero, i.e., Error polynomial.

Probability

Next up?

Probability

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A bag contains:

Probability

Next up? Probability.

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Probability

Next up? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Probability

Next up? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue.

Probability

Next up? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total.

Probability

Next up? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide.

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For now:

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Later: Probability.