Today

Random Variables.

Review:Poll.

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C is definition of conditional probability

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1. Random Variables.
2. Expectation
3. Distributions.

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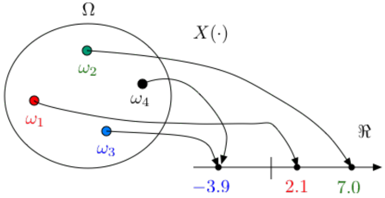

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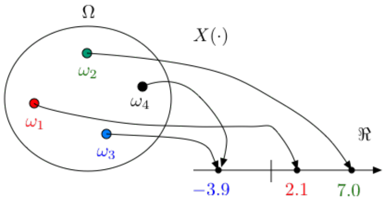

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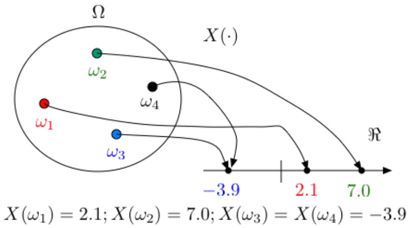

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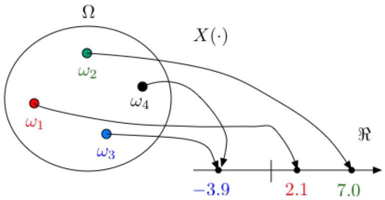

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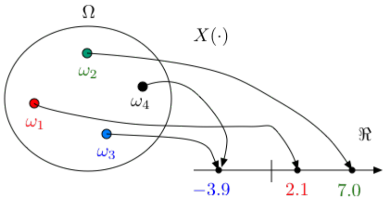

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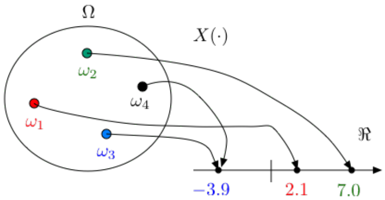

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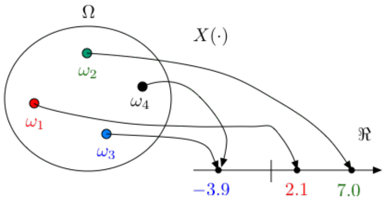

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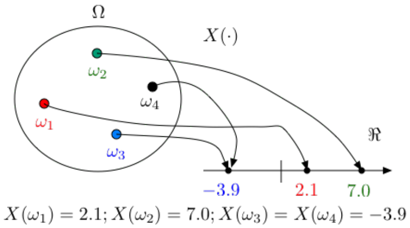

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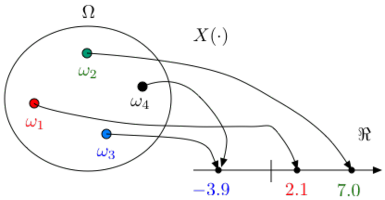
Thus, $X(\cdot)$ assigns a real number $X(\omega)$ to each $\omega \in \Omega$.



$X(\omega_1) = 2.1; X(\omega_2) = 7.0; X(\omega_3) = X(\omega_4) = -3.9$

Function $X(\cdot)$ defined on outcomes Ω.

Function $X(\cdot)$ is not random, not a variable!

What varies at random (among experiments)? The outcome!

Random variable makes partition: $A_y = \{\omega \in \Omega : X(\omega) = y\} = X^{-1}(y)$
$A_{-3.9} = \{\omega_3, \omega_4\}, A_{2.1} = \{\omega_1\}, A_{7.0} = \{\omega_2\}$.

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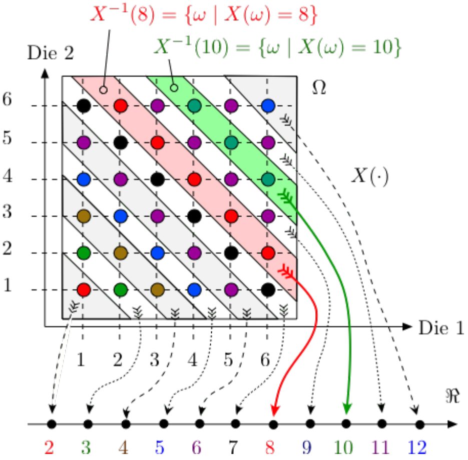

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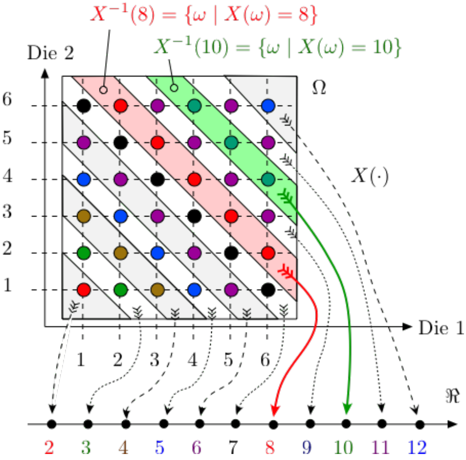

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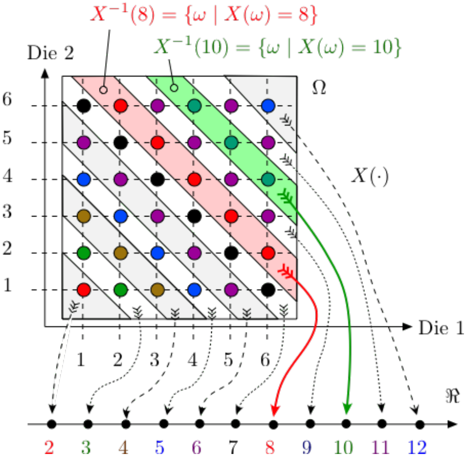

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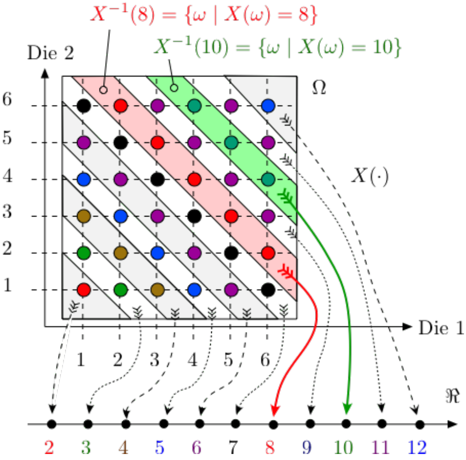

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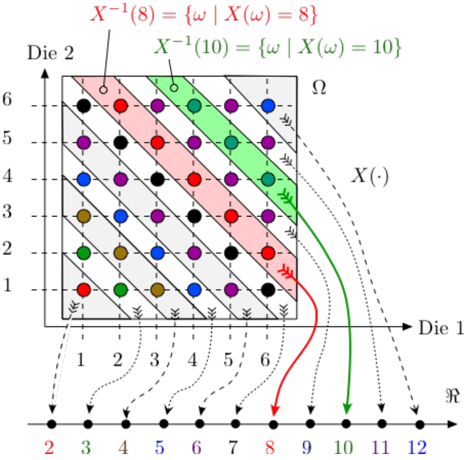

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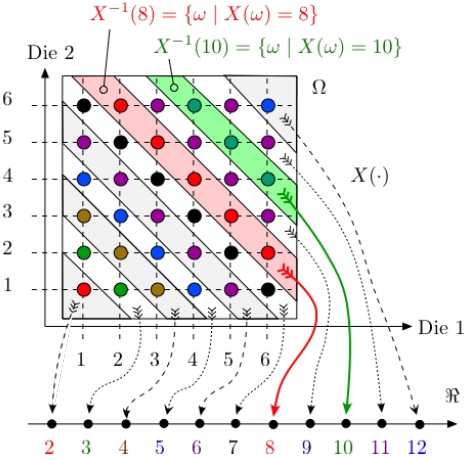

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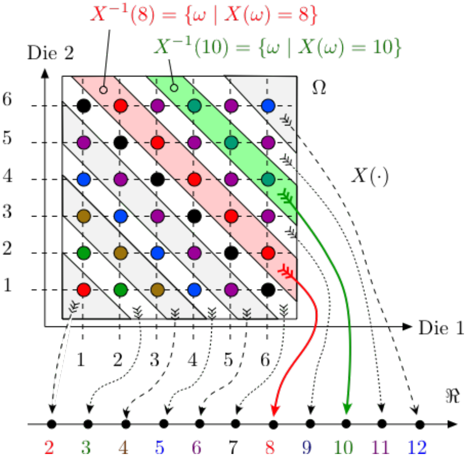

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The probability of X taking on a value a.

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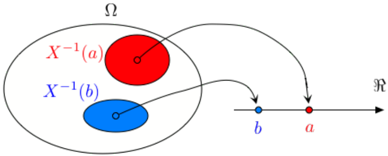

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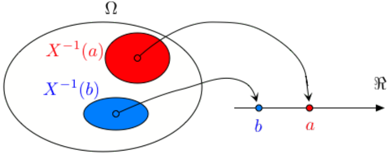

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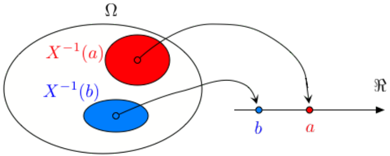

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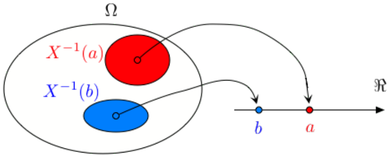

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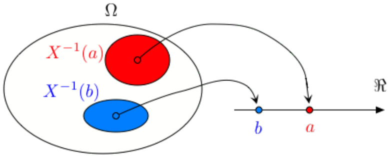

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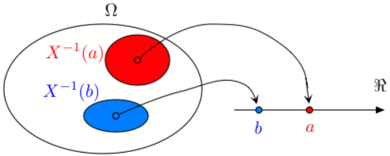

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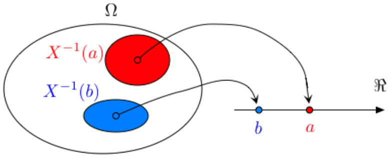

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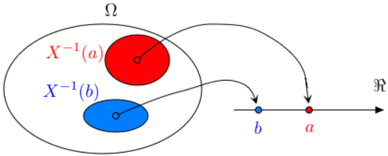

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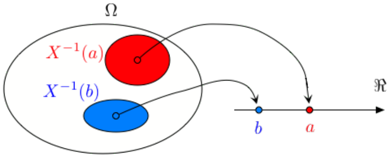

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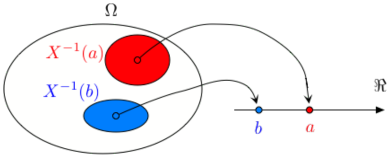

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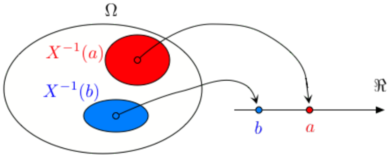

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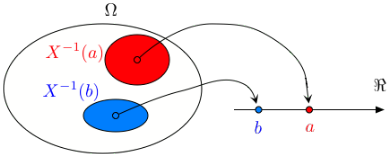

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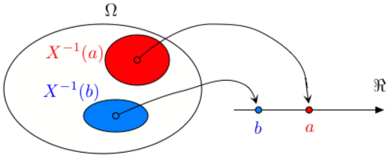

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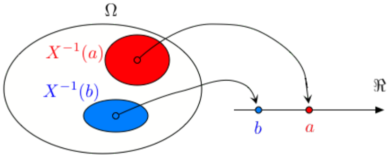

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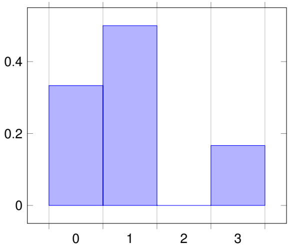

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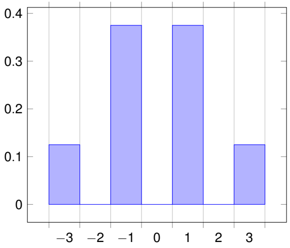

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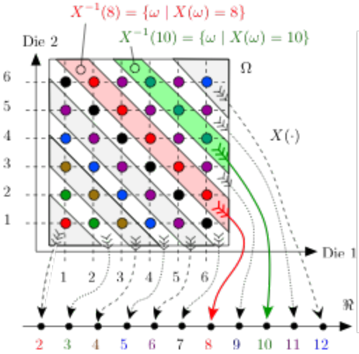

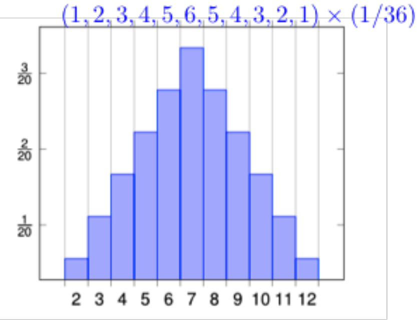

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We use this frequentist interpretation as a definition.

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The subjectivist(bayesian) interpretation of $E[X]$ is less obvious.

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Distributive property of multiplication over addition.

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The binomial distribution: Poll.

Flip n coins with heads probability p.

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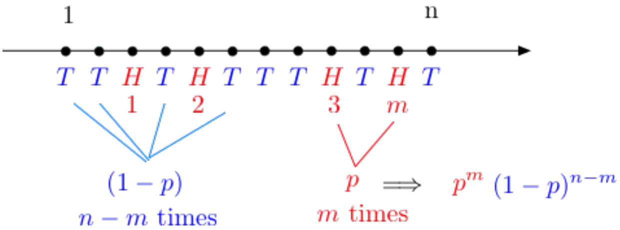

$\binom{n}{m}$ outcomes with m Hs and $n - m$ Ts

$$\Longrightarrow \boxed{Pr[X = m] = \binom{n}{m} p^m (1-p)^{n-m}}$$

Error channel and...

A packet is corrupted with probability p.

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Also distribution in polling, experiments, etc.

Expectation of Binomial Distibution

Parameter p and n. What is the expectation?

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Waiting is good.

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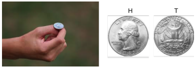

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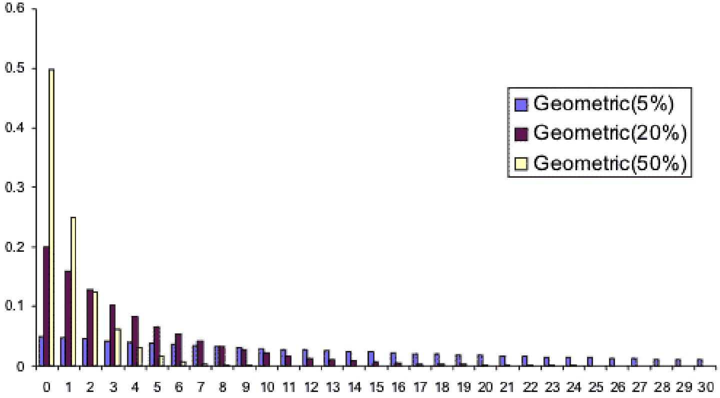

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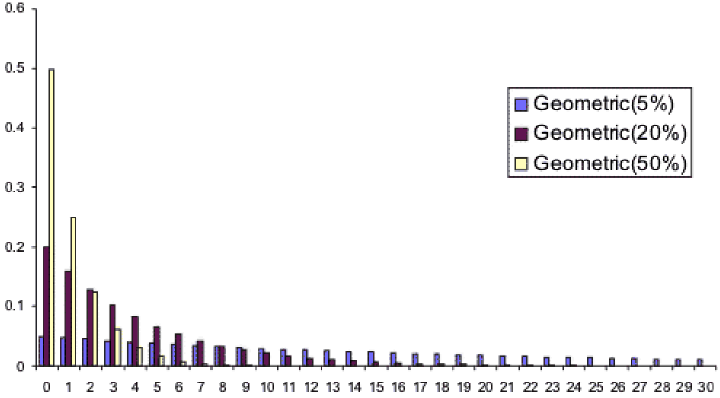

Oops: $Pr[X - 1]$.

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Poisson: Motivation and derivation.

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Model with binomial.

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Experiment: flip a coin *n* times.

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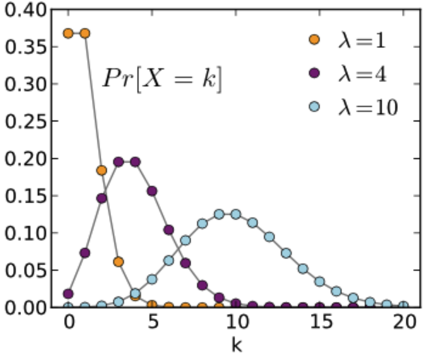

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For (1) we used $m \ll n$; for (2) we used $(1 - a/n)^n \approx e^{-a}$.

Poisson Distribution: Definition and Mean

Definition Poisson Distribution with parameter $\lambda > 0$

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Fact: $E[X] = \lambda$.

Proof:

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Poisson Distribution: Definition and Mean

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Second line: Taylor's expansion of e^{λ}. $\square$

Simeon Poisson

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Siméon Poisson

Siméon Denis Poisson (1781–1840)

Equal Time: B. Geometric

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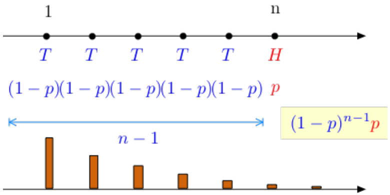

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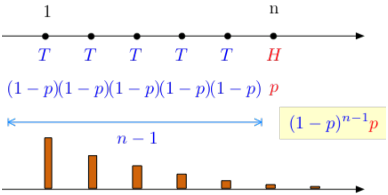

I could not find a picture of D. Binomial, sorry.

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- $B(n,p), U[1:n], G(p), P(\lambda)$.