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Propositional forms correspond to truth tables.

Logical equivalence of forms means same truth tables.

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Wait!

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Wait! What is $\mathbb{N}$?

Quantifiers: universes.

Proposition: "For all natural numbers n**,** $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$**."**

Proposition has **universe**:

Quantifiers: universes.

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Proposition has **universe**: "the natural numbers".

Quantifiers: universes.

Proposition: "For all natural numbers n, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$."

Proposition has **universe**: "the natural numbers".

Universe examples include..

- $\mathbb{N} = \{0, 1, \ldots\}$ (natural numbers).
- $\mathbb{Z} = \{\ldots, -1, 0, \ldots\}$ (integers)
- $\mathbb{Z}^+$ (positive integers)
- $\mathbb{R}$ (real numbers)
- Any set: $S = \{Alice, Bob, Charlie, Donna\}$.
- See note 0 for more!

Back to: Wason's experiment:1

Theory:

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Statement/theory: $\forall x \in \{A, B, C, D\}, Chicago(x)$

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So $Chicago(Bob)$ must be False .

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No. $Chicago(D) \implies Flew(D)$ is true if $Flew(D)$ is true.

Only have to turn over cards for Bob and Charlie.

More for all quantifiers examples.

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- “doubling a number always makes it larger”

More for all quantifiers examples.

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$$(\forall x \in \mathbb{N})\,(2x > x)$$

More for all quantifiers examples.

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Later we may omit universe if clear from context.

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for $n = 2$, we have 3,4,5 and 5,7, 12 and ...

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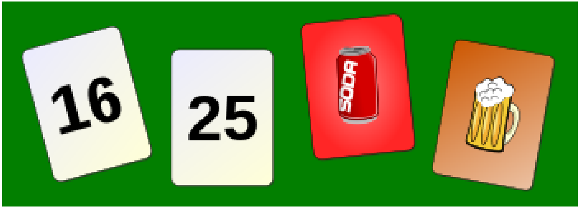
And now: proofs!

Review.



Theory: If you drink alcohol you must be at least 18.

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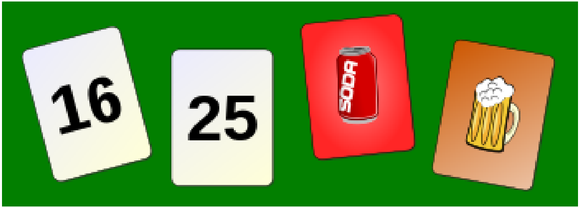

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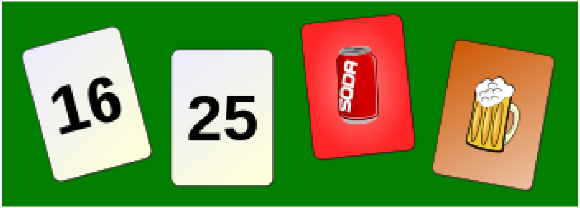

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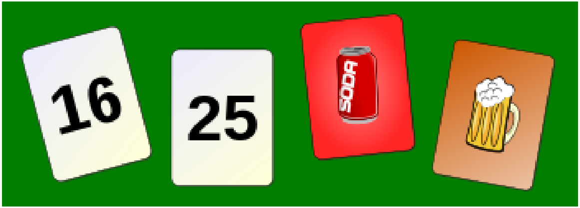

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"< 18" $\implies$ Don't Drink Alcohol.

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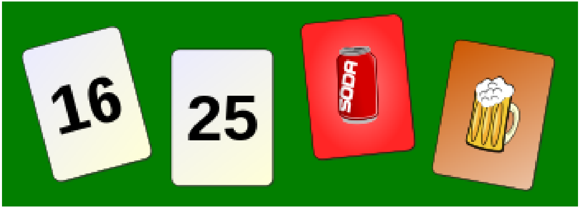

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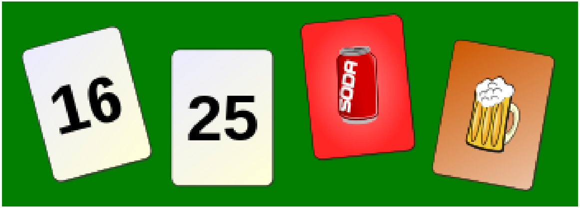

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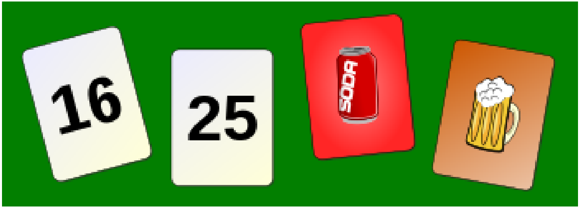

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CS70: Lecture 2. Outline.

Today: Proofs!!!

1. By Example.
2. Direct. (Prove $P \implies Q$.)
3. by Contraposition (Prove $P \implies Q$)
4. by Contradiction (Prove P.)
5. by Cases

If time: discuss induction.

Last time: Existential statement.

How to prove existential statement?

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□

Often used to disprove claim.

Quick Background, Notation and *Definitions!*

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Divides.

$a|b$ means

(A) There exists $k \in \mathbb{Z}$, with $a = kb$.

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Correct: (B) and (E).

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Theorem: For any $a, b, c \in Z$, if $a|b$ and $a|c$ then $a|(b-c)$.

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Direct Proof Form:
Goal: $P \implies Q$
Assume P.
...
Therefore Q.

Another direct proof.

Let D_3 be the 3 digit natural numbers.

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$n = 121$

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$n = 121$ Alt Sum: $1 - 2 + 1 = 0$.

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Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

$n = 605$ Alt Sum: $6 - 0 + 5 = 11$

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$n = 605$ Alt Sum: $6 - 0 + 5 = 11$ Divis. by 11. As is $605 = 11(55)$

Proof: For $n \in D_3$, $n = 100a + 10b + c$, for some a, b, c.

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Examples:

$n = 121$ Alt Sum: $1 - 2 + 1 = 0$. Divis. by 11. As is 121.

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/ˌäbfəˈskāSH(ə)n/

noun

noun: **obfuscation**; plural noun: **obfuscations**

the action of making something obscure, unclear, or unintelligible.

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Did we prove?

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  As it assumed the only primes were the first k primes.

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Proof by cases.

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Question: Which case holds?

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Question: Which case holds? Don't know!!!

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Which of the following are (certainly) true?

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Which of the following are (certainly) true?

(A) $\sqrt{2}$ is irrational.

(B) $\sqrt{2}^{\sqrt{2}}$ is rational.

(C) $\sqrt{2}^{\sqrt{2}}$ is rational or it isn't.

(D) $(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}}$ is rational.

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(C) $\sqrt{2}^{\sqrt{2}}$ is rational or it isn't.

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(A),(C),(D)

Poll: proof review.

Which of the following are (certainly) true?

(A) $\sqrt{2}$ is irrational.

(B) $\sqrt{2}^{\sqrt{2}}$ is rational.

(C) $\sqrt{2}^{\sqrt{2}}$ is rational or it isn't.

(D) $(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}}$ is rational.

(A),(C),(D)

(B) I don't know.

Be careful.

Theorem: $3 = 4$

Be careful.

Theorem: $3 = 4$

Proof: Assume $3 = 4$.

Be careful.

Theorem: $3 = 4$

Proof: Assume $3 = 4$.

Start with $12 = 12$.

Be careful.

Theorem: $3 = 4$

Proof: Assume $3 = 4$.

Start with $12 = 12$.

Divide one side by 3 and the other by 4 to get
$$4 = 3.$$

Be careful.

Theorem: $3 = 4$

Proof: Assume $3 = 4$.

Start with $12 = 12$.

Divide one side by 3 and the other by 4 to get
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By commutativity

Be careful.

Theorem: $3 = 4$

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Divide one side by 3 and the other by 4 to get
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By commutativity theorem holds.

Be careful.

Theorem: $3 = 4$

Proof: Assume $3 = 4$.

Start with $12 = 12$.

Divide one side by 3 and the other by 4 to get
$$4 = 3.$$

By commutativity theorem holds. $\square$

Be careful.

Theorem: $3 = 4$

Proof: Assume $3 = 4$.

Start with $12 = 12$.

Divide one side by 3 and the other by 4 to get
$4 = 3$.

By commutativity theorem holds. $\square$

What's wrong?

Be careful.

Theorem: $3 = 4$

Proof: Assume $3 = 4$.

Start with $12 = 12$.

Divide one side by 3 and the other by 4 to get
$4 = 3$.

By commutativity theorem holds. $\square$

What's wrong?

Don't assume what you want to prove!

Be really careful!

Theorem: $1 = 2$
Proof:

Be really careful!

Theorem: $1 = 2$

Proof: For $x = y$, we have

Be really careful!

Theorem: $1 = 2$

Proof: For $x = y$, we have

$$(x^2 - xy) = x^2 - y^2$$

Be really careful!

Theorem: $1 = 2$

Proof: For $x = y$, we have

$$(x^2 - xy) = x^2 - y^2$$
$$x(x - y) = (x + y)(x - y)$$

Be really careful!

Theorem: $1 = 2$

Proof: For $x = y$, we have

$$\begin{aligned} (x^2 - xy) &= x^2 - y^2 \\ x(x - y) &= (x + y)(x - y) \\ x &= (x + y) \end{aligned}$$

Be really careful!

Theorem: $1 = 2$

Proof: For $x = y$, we have

$$\begin{aligned} (x^2 - xy) &= x^2 - y^2 \\ x(x-y) &= (x+y)(x-y) \\ x &= (x+y) \\ x &= 2x \end{aligned}$$

Be really careful!

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Poll: What is the problem?

(A) Assumed what you were proving.

(B) No problem. Its fine.

(C) $x - y$ is zero.

(D) Can't multiply by zero in a proof.

Be really careful!

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Dividing by zero is no good.

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Dividing by zero is no good. Multiplying by zero is wierdly cool!

Be really careful!

Theorem: $1 = 2$

Proof: For $x = y$, we have

$$(x^2 - xy) = x^2 - y^2$$
$$x(x - y) = (x + y)(x - y)$$
$$x = (x + y)$$
$$x = 2x$$
$$1 = 2$$

□

Poll: What is the problem?

(A) Assumed what you were proving.

(B) No problem. Its fine.

(C) $x - y$ is zero.

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Dividing by zero is no good. Multiplying by zero is wierdly cool!

Also: Multiplying inequalities by a negative.

Be really careful!

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Dividing by zero is no good. Multiplying by zero is wierdly cool!

Also: Multiplying inequalities by a negative.

$P \implies Q$ does not mean $Q \implies P$.

Summary: Note 2.

Direct Proof:

Summary: Note 2.

Direct Proof:

To Prove: $P \implies Q$.

Summary: Note 2.

Direct Proof:
To Prove: $P \implies Q$. Assume P.

Summary: Note 2.

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To Prove: $P \implies Q$. Assume P. Prove Q.

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Summary: Note 2.

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$a|b$ and $a|c \implies a|(b-c)$.

By Contraposition:

To Prove: $P \implies Q$ Assume $\neg Q$.

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Direct Proof:

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$a|b$ and $a|c \implies a|(b-c)$.

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n^2 is odd $\implies n$ is odd.

Summary: Note 2.

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To Prove: P

Summary: Note 2.

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By Contradiction:

To Prove: P Assume $\neg P$. Prove False .

$\sqrt{2}$ is rational.

Summary: Note 2.

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By Cases: informal.

Summary: Note 2.

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Universal: show that statement holds in all cases.

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Existence: used cases where one is true.

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Either $\sqrt{2}$ and $\sqrt{2}$ worked.

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Either $\sqrt{2}$ and $\sqrt{2}$ worked.

or $\sqrt{2}$ and $\sqrt{2}^{\sqrt{2}}$ worked.

Summary: Note 2.

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To Prove: $P \implies Q$. Assume P. Prove Q.

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Careful when proving!

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To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

n^2 is odd $\implies n$ is odd. $\equiv n$ is even $\implies n^2$ is even.

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Existence: used cases where one is true.

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or $\sqrt{2}$ and $\sqrt{2}^{\sqrt{2}}$ worked.

Careful when proving!

Don't assume the theorem.

Summary: Note 2.

Direct Proof:

To Prove: $P \implies Q$. Assume P. Prove Q.

$a|b$ and $a|c \implies a|(b-c)$.

By Contraposition:

To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

n^2 is odd $\implies$ n is odd. $\equiv$ n is even $\implies$ n^2 is even.

By Contradiction:

To Prove: P Assume $\neg P$. Prove False .

$\sqrt{2}$ is rational.

$\sqrt{2} = \frac{a}{b}$ with no common factors....

By Cases: informal.

Universal: show that statement holds in all cases.

Existence: used cases where one is true.

Either $\sqrt{2}$ and $\sqrt{2}$ worked.

or $\sqrt{2}$ and $\sqrt{2}^{\sqrt{2}}$ worked.

Careful when proving!

Don't assume the theorem. Divide by zero.

Summary: Note 2.

Direct Proof:

To Prove: $P \implies Q$. Assume P. Prove Q.

$a|b$ and $a|c \implies a|(b-c)$.

By Contraposition:

To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

n^2 is odd $\implies$ n is odd. $\equiv$ n is even $\implies$ n^2 is even.

By Contradiction:

To Prove: P Assume $\neg P$. Prove False .

$\sqrt{2}$ is rational.

$\sqrt{2} = \frac{a}{b}$ with no common factors....

By Cases: informal.

Universal: show that statement holds in all cases.

Existence: used cases where one is true.

Either $\sqrt{2}$ and $\sqrt{2}$ worked.

or $\sqrt{2}$ and $\sqrt{2}^{\sqrt{2}}$ worked.

Careful when proving!

Don't assume the theorem. Divide by zero.Watch converse.

Summary: Note 2.

Direct Proof:

To Prove: $P \implies Q$. Assume P. Prove Q.

$a|b$ and $a|c \implies a|(b-c)$.

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To Prove: $P \implies Q$ Assume $\neg Q$. Prove $\neg P$.

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$\sqrt{2}$ is rational.

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Either $\sqrt{2}$ and $\sqrt{2}$ worked.

or $\sqrt{2}$ and $\sqrt{2}^{\sqrt{2}}$ worked.

Careful when proving!

Don't assume the theorem. Divide by zero.Watch converse. ...

CS70: Note 3. Induction!

Poll. What's the biggest number?

(A) 100

(B) 101

(C) n+1

(D) infinity.

(E) This is about the "recursive leap of faith."