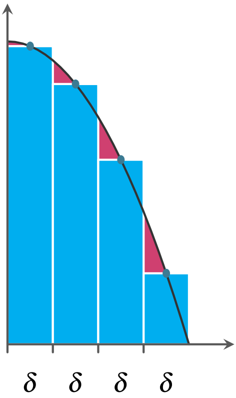
Survey

Fill it out!!
tinyurl.com/cs70-survey

Calculus



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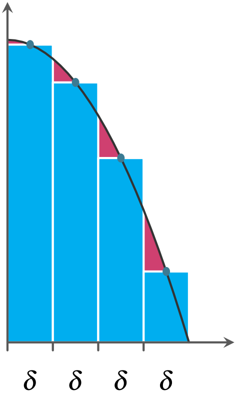

Riemann Sum/Integral: $\int_a^b f(x)dx = lim_{\delta \to 0} \sum_i \delta f(a_i)$

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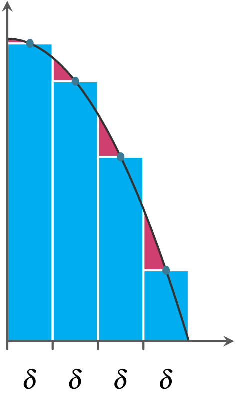

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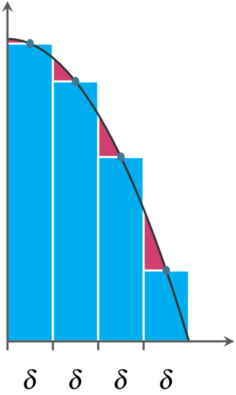

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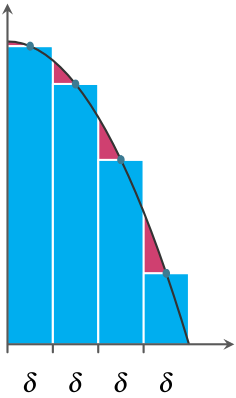

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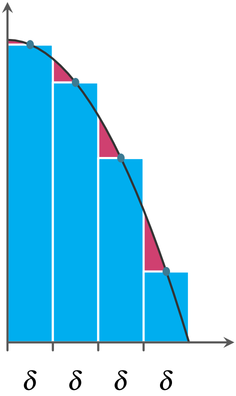

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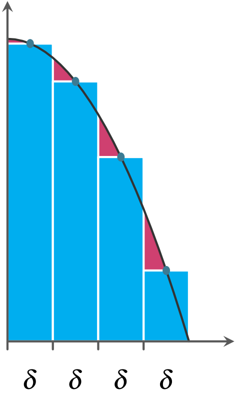

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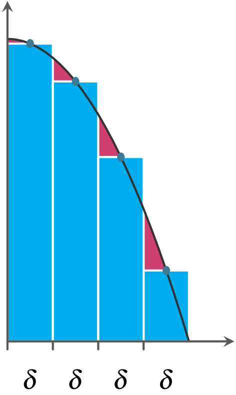

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Thus $F'(x) = f(x)$.

CS70: Continuous Probability.

Continuous Probability 1

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Continuous Probability 1

1. Examples
2. Events
3. Continuous Random Variables

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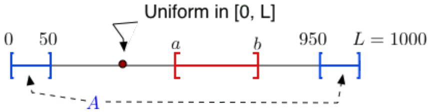

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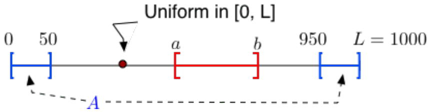

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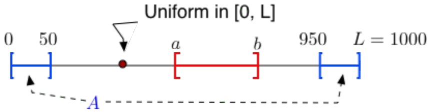

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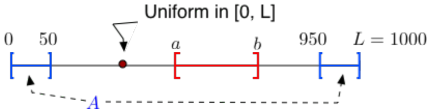

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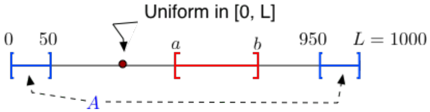

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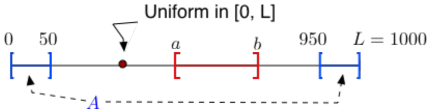

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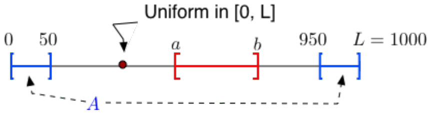

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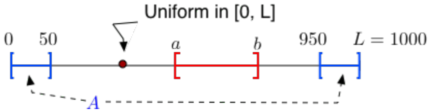

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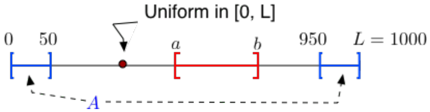

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Makes sense: $b - a$ is the fraction of $[0,1]$ that $[a,b]$ covers.

Poll

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Idea. Tail Sum. For positive integer valued X, $E[X] = \sum_i Pr[X \geq i]$.

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$$Pr[\cup_n A_n] := \sum_n Pr[A_n].$$

Many subsets of $[0,1]$ are of this form.

Uniformly at Random in $[0,1]$.

Let $[a,b]$ denote the **event** that the point X is in the interval $[a,b]$.

$$Pr[[a,b]] = \frac{\text{length of } [a,b]}{\text{length of } [0,1]} = \frac{b-a}{1} = b-a.$$

Intervals like $[a,b] \subseteq \Omega = [0,1]$ are **events.**
More generally, events in this space are unions of intervals.
Example: the event A - "within 0.2 of 0 or 1" is $A = [0,0.2] \cup [0.8,1]$.
Thus,

$$Pr[A] = Pr[[0,0.2]] + Pr[[0.8,1]] = 0.4.$$

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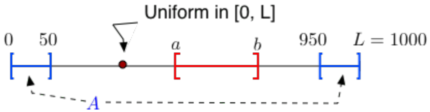

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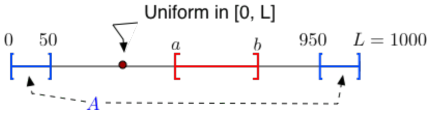
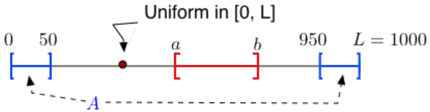


Note: A **radical** change in approach.

Uniformly at Random in $[0, 1]$.



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Finite prob. space:

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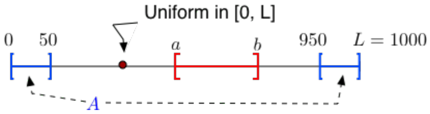

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$\Omega = \{1, 2, \ldots, N\}$,

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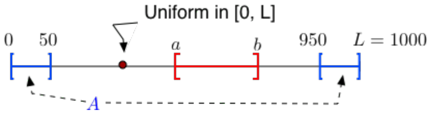

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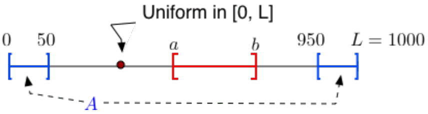

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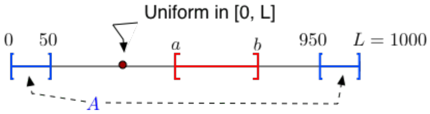

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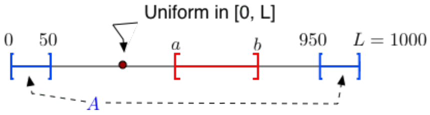

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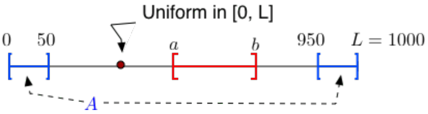

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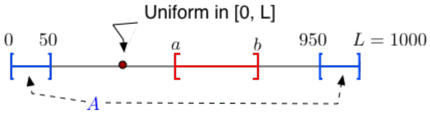

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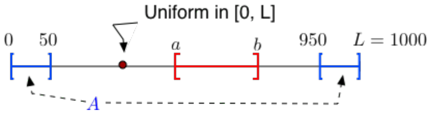

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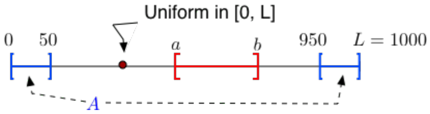

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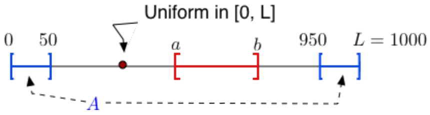

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Event A = interval,

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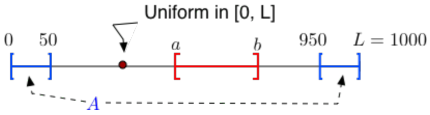

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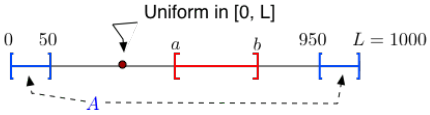

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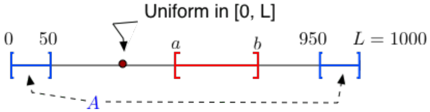

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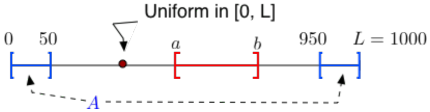

$Pr[X \leq x] = x$ for $x \in [0, 1]$.

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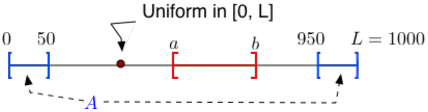

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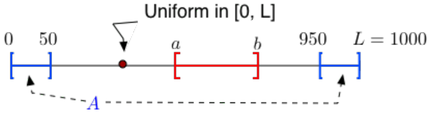

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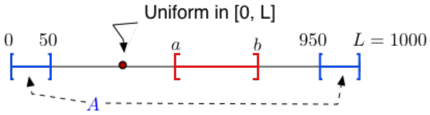

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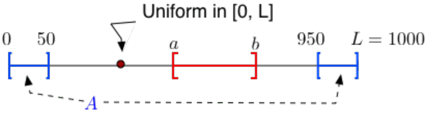

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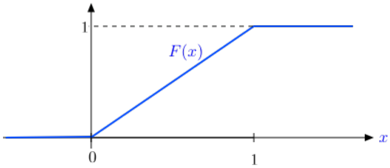

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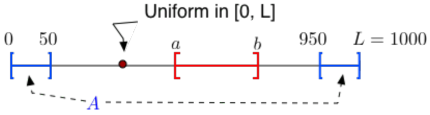

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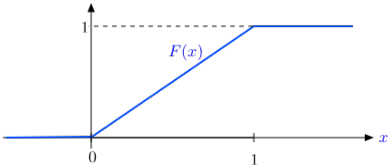

Then we have $Pr[X \in (a,b]] = Pr[X \le b] - Pr[X \le a]$

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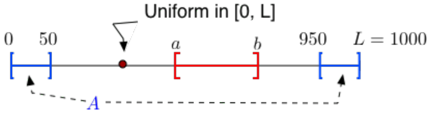

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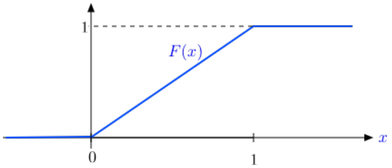

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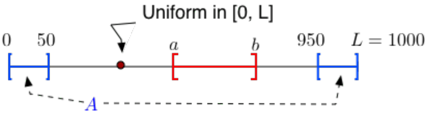

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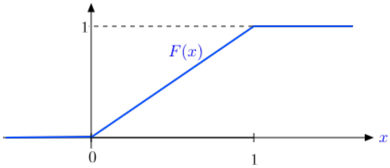
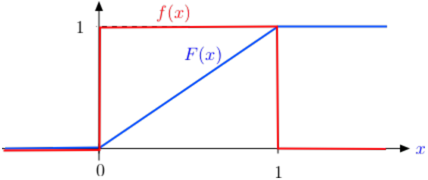


Then we have $Pr[X \in (a,b]] = Pr[X \leq b] - Pr[X \leq a] = F(b) - F(a)$.

Thus, $F(\cdot)$ specifies the probability of all the events!

Uniformly at Random in $[0, 1]$.



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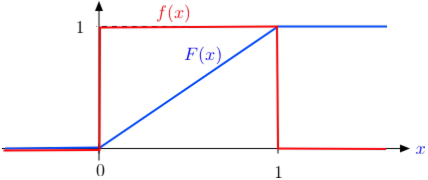

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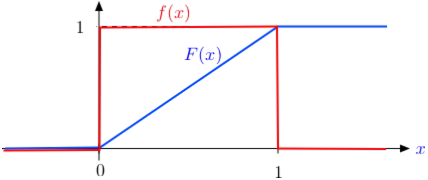

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Alternative view is *density function* $f(x) = \frac{d}{dx}F(x) =$

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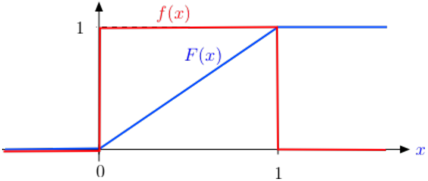

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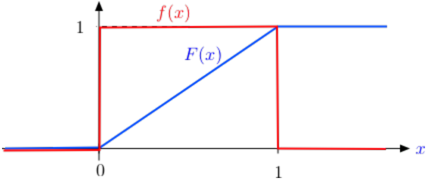

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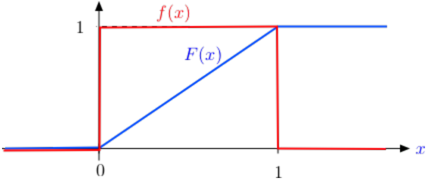

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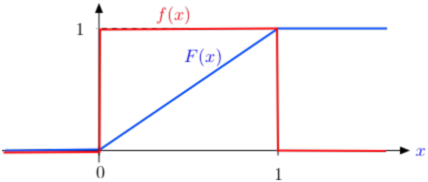

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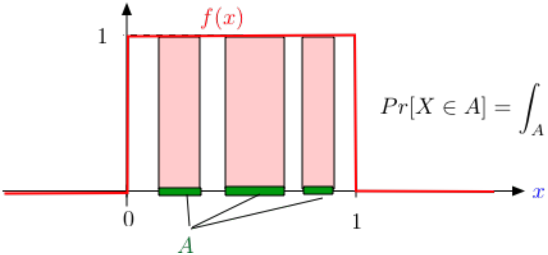

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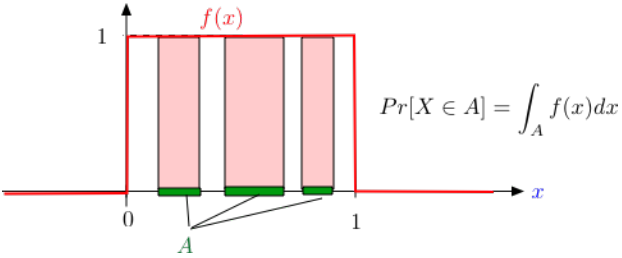
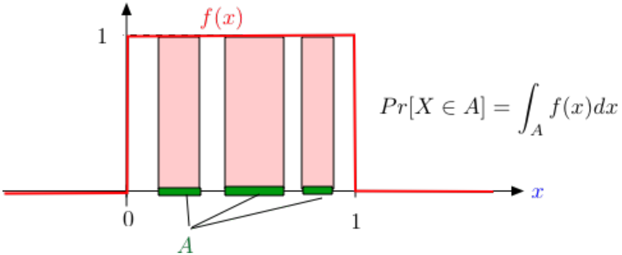


Think of $f(x)$ as describing how
one unit of probability is spread over $[0, 1]$:

Uniformly at Random in $[0, 1]$.



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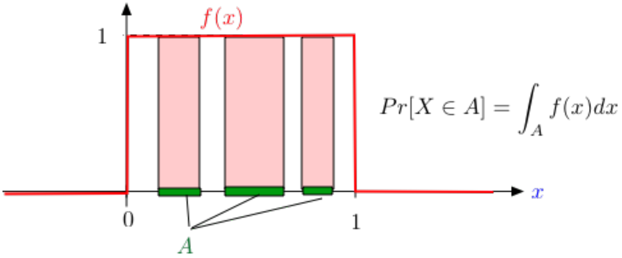

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Then $Pr[X \in A]$ is the probability mass over A.

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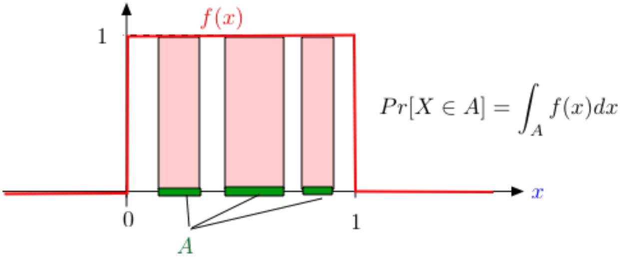

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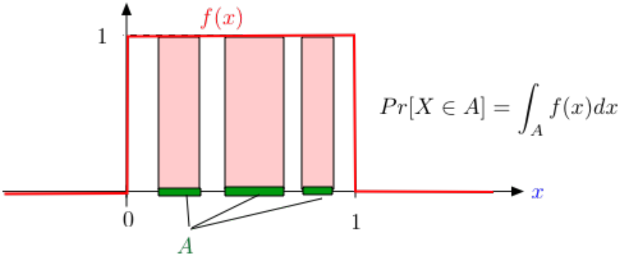

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Observe:

- This makes the probability automatically additive.

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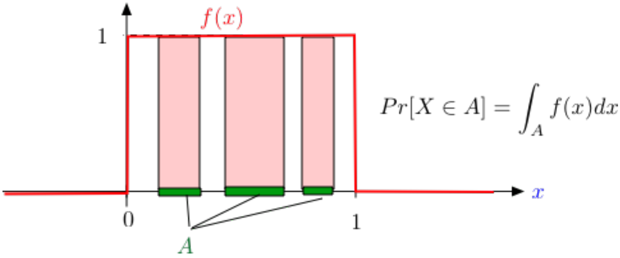

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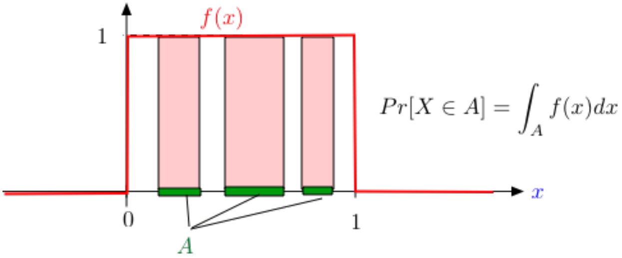
- This makes the probability automatically additive.
- We need $f(x) \geq 0$

Uniformly at Random in $[0, 1]$.



Think of $f(x)$ as describing how
one unit of probability is spread over $[0, 1]$: uniformly!

Then $Pr[X \in A]$ is the probability mass over A.

Observe:

- This makes the probability automatically additive.
- We need $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x)dx = 1$.

Uniformly at Random in $[0, 1]$.

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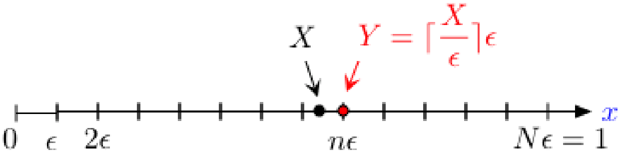

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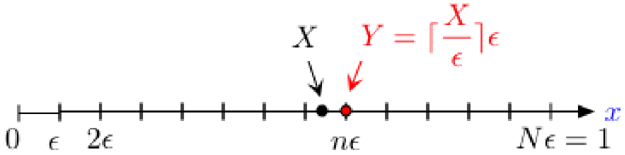

Discrete Approximation:

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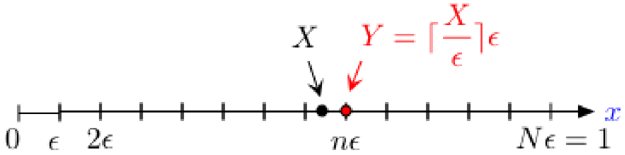

Discrete Approximation: Fix $N \gg 1$

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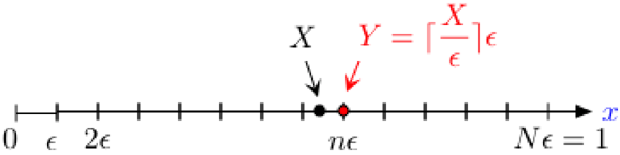
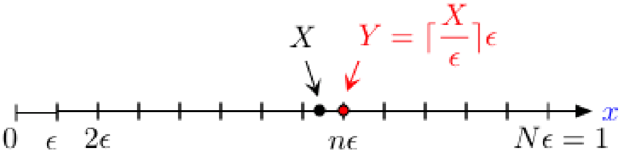


Discrete Approximation: Fix $N \gg 1$ and let $\varepsilon = 1/N$.

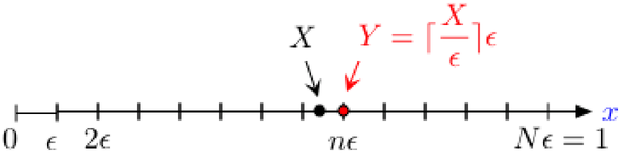
Uniformly at Random in $[0,1]$.



Discrete Approximation: Fix $N \gg 1$ and let $\varepsilon = 1/N$.

Define $Y = n\varepsilon$ if $(n-1)\varepsilon < X \leq n\varepsilon$ for $n = 1, \ldots, N$.

Uniformly at Random in $[0,1]$.



Discrete Approximation: Fix $N \gg 1$ and let $\varepsilon = 1/N$.

Define $Y = n\varepsilon$ if $(n-1)\varepsilon < X \leq n\varepsilon$ for $n = 1, \ldots, N$.

Then $|X - Y| \leq \varepsilon$

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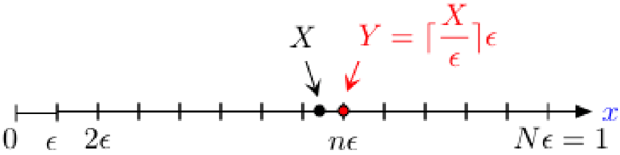

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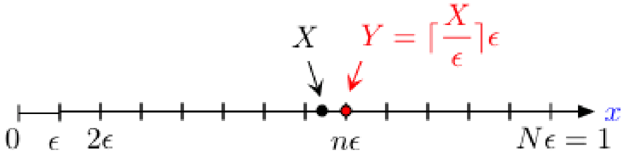
Define $Y = n\varepsilon$ if $(n-1)\varepsilon < X \leq n\varepsilon$ for $n = 1, \ldots, N$.

Then $|X - Y| \leq \varepsilon$ and Y is discrete:

Uniformly at Random in $[0,1]$.



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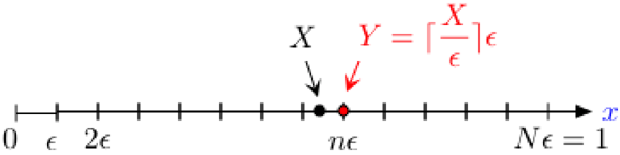

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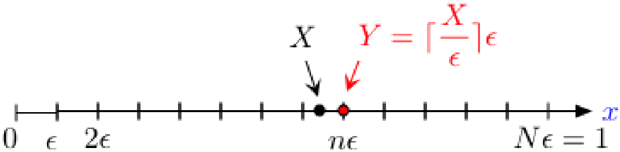

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Thus, X is 'almost discrete.'

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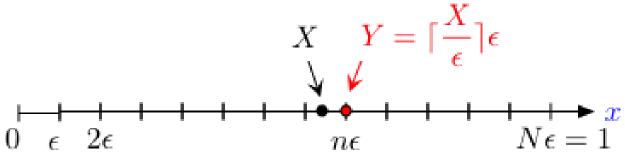

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Calculus view: $Pr[Y = n\varepsilon]$ is area of rectangle in Riemann sum.

Nonuniformly at Random in $[0, 1]$.

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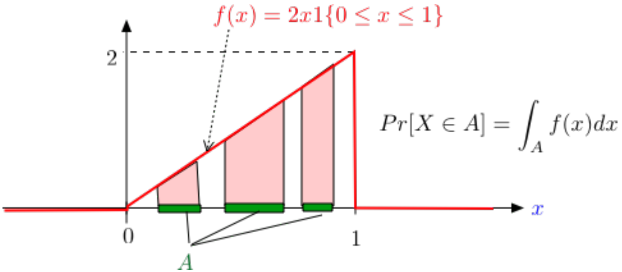

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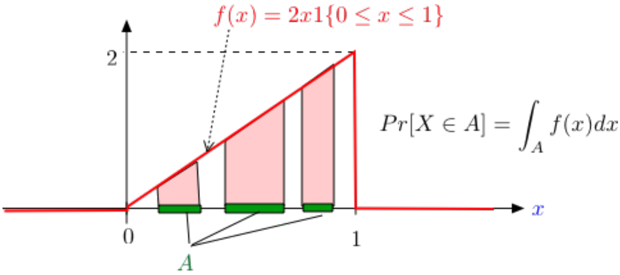

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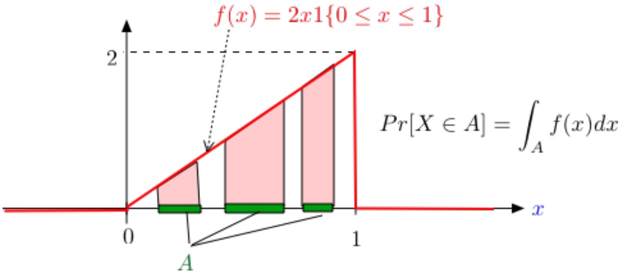

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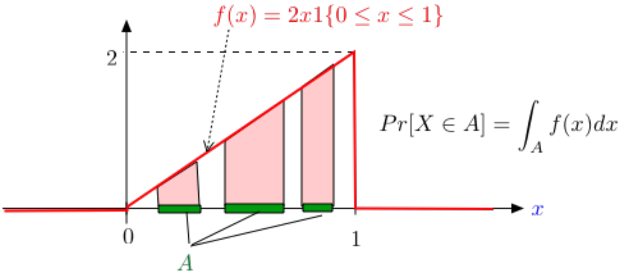

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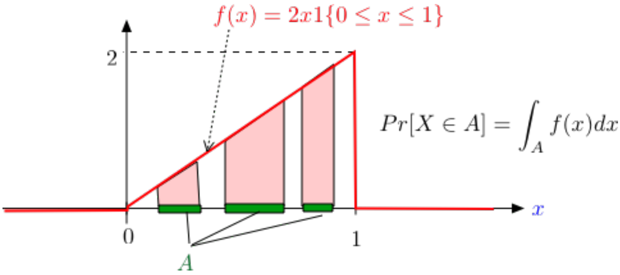

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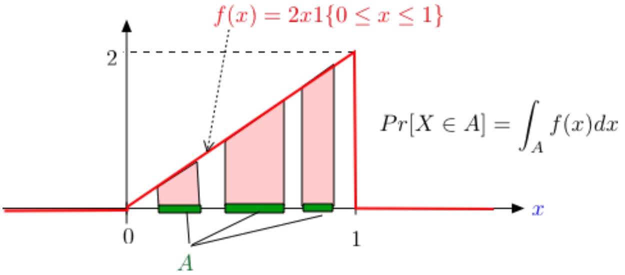

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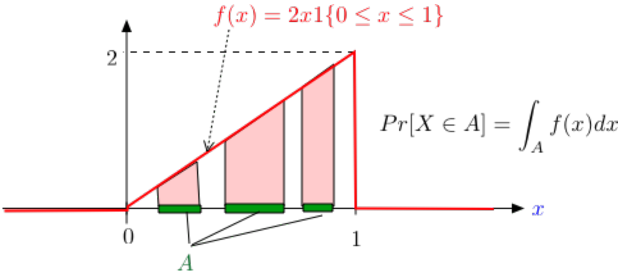

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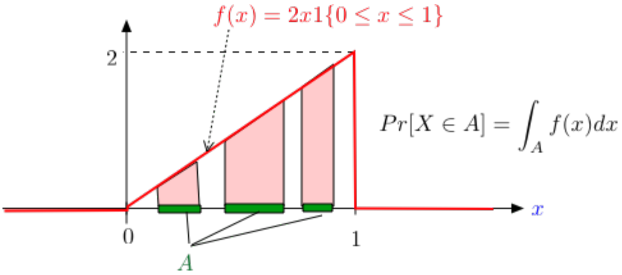

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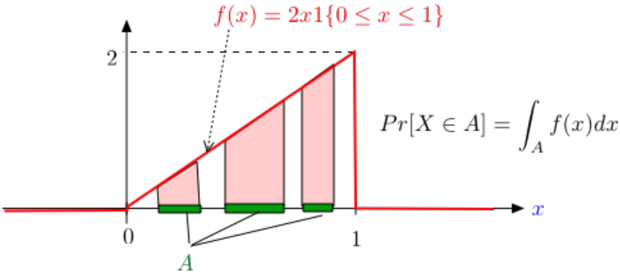

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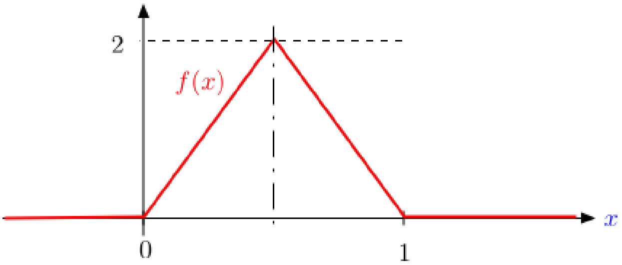

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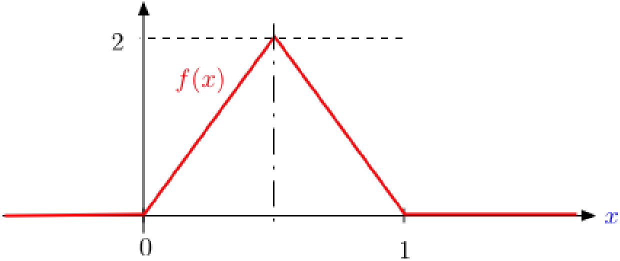

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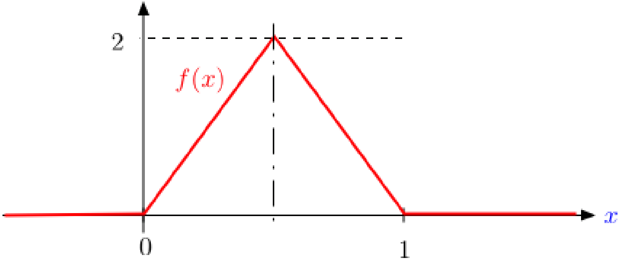

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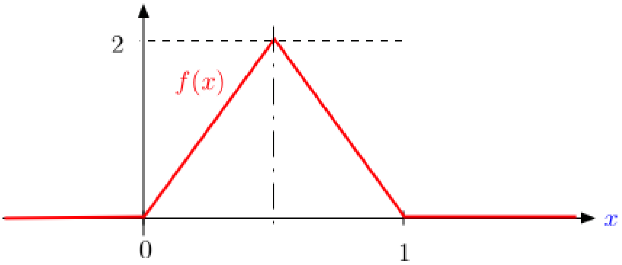

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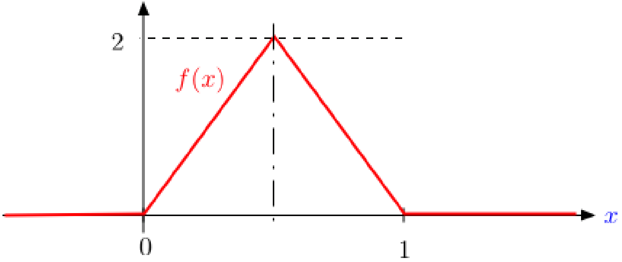

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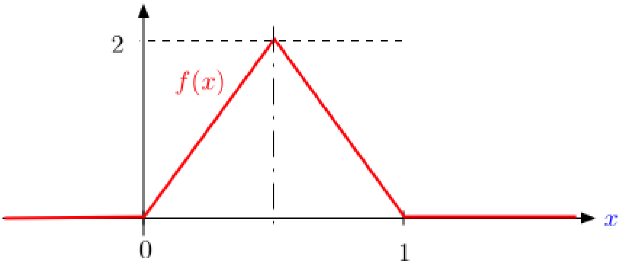

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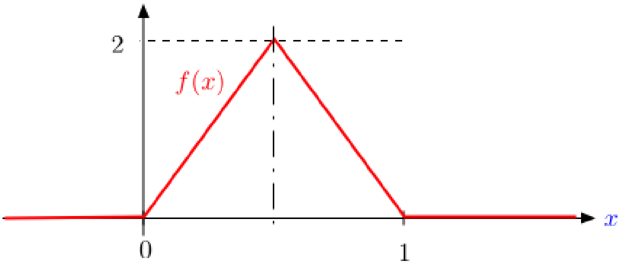

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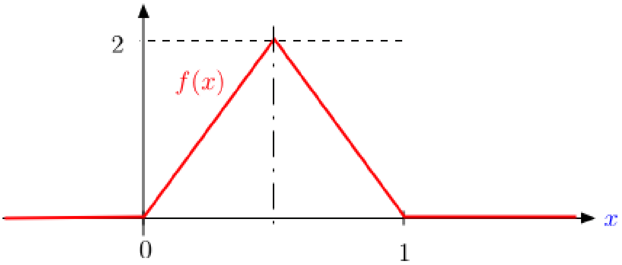

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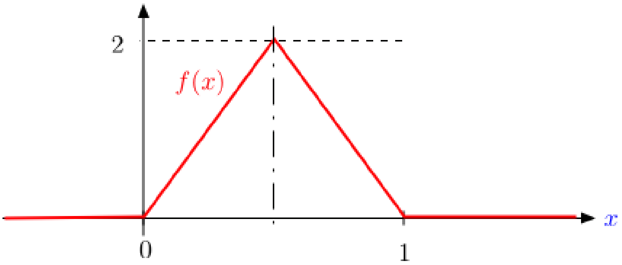

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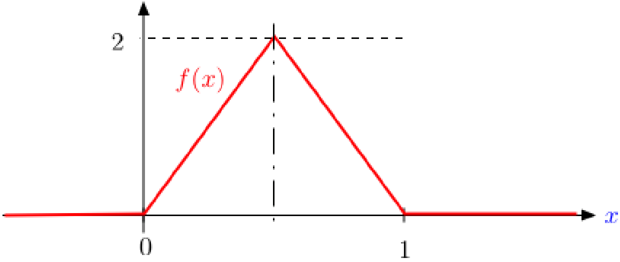

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General Random Choice in $\Re$

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Let $F(x)$ be a nondecreasing function

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When F and f correspond RV X: $F_X(x)$ and $f_X(x)$.

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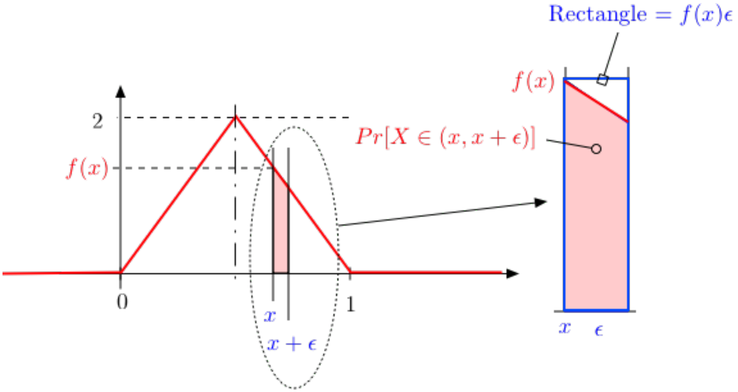

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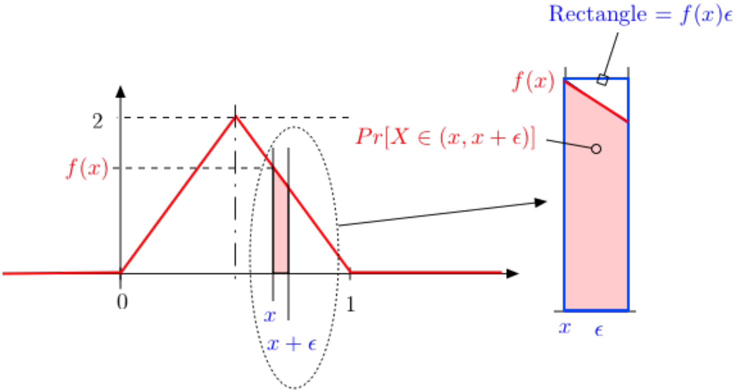

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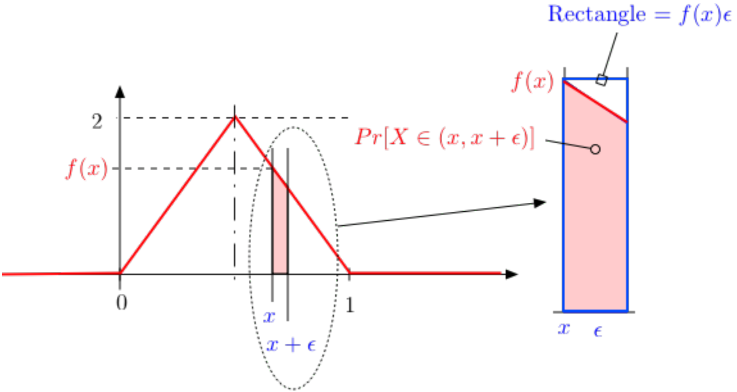

Thus, the pdf is the 'local probability by unit length.'

It is the 'probability density.'

Discrete Approximation

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Example: CDF, pre-poll

Example: hitting random location on gas tank.

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Example: hitting random location on gas tank.
Random location on circle.

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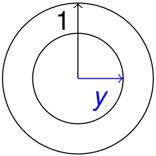

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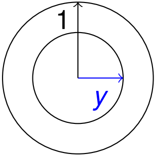

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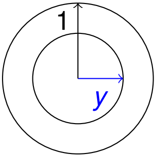

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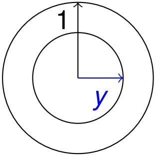

Random Variable: Y distance from center.

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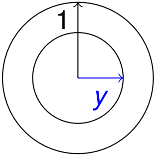

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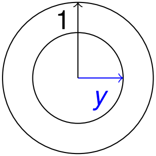

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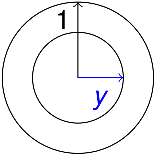

Random Variable: Y distance from center.
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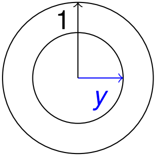

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Hence,

$$F_Y(y) = Pr[Y \leq y] = \begin{cases} 0 & \text{for } y < 0 \\ y^2 & \text{for } 0 \leq y \leq 1 \\ 1 & \text{for } y > 1 \end{cases}$$

Calculation of event with dartboard..

Probability between .5 and .6 of center?

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PDF.

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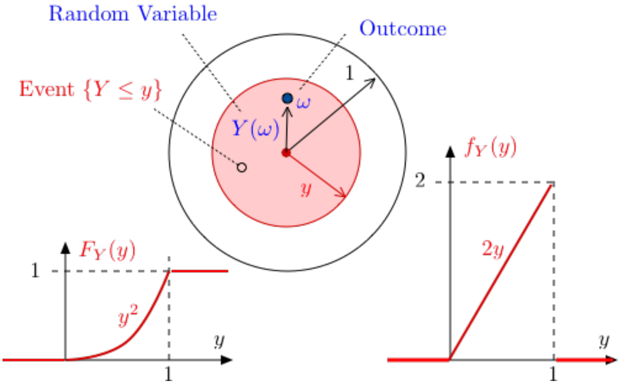
The cumulative distribution function (cdf) and probability distribution function (pdf) give full information.

Use whichever is convenient.

Target

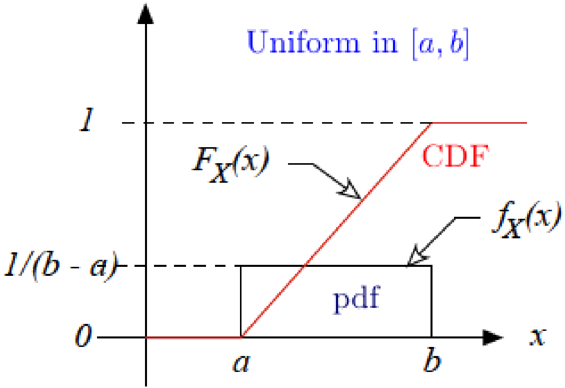
Target



Random Variable
Outcome
Event $\{Y \leq y\}$
1
ω
$Y(\omega)$
y
$f_Y(y)$
2
$2y$
$F_Y(y)$
1
y^2
y
1
y
1

$U[a,b]$

$U[a, b]$



Exponential derivation:Poll.

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What is true?

(A) $X \sim G(p)$
(B) $Pr[X > i] = (1-p)^i$.
(C) $Pr[Y > i/n] = (1-\lambda/n)^i$.
(D) $Pr[Y > y] = (1-\lambda/n)^{ny}$.
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(E) $(1-\lambda/n)^{ny} = ((1-\lambda/n)^{n/\lambda})^{\lambda y}$
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The limit as $n \to \infty$ of Y has $Pr[Y > y] = e^{-\lambda y}$.

Exponential derivation:Poll.

$Pr[X = i] = (1 - p)^{i-1}p$.

Let $p = \lambda/n$. and $Y = X/n$.

What is true?

(A) $X \sim G(p)$
(B) $Pr[X > i] = (1 - p)^i$.
(C) $Pr[Y > i/n] = (1 - \lambda/n)^i$.
(D) $Pr[Y > y] = (1 - \lambda/n)^{ny}$.
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(A) True by definition.
(B) $Pr[X > i] = (1 - p)^i$ at least i coin flips fail.
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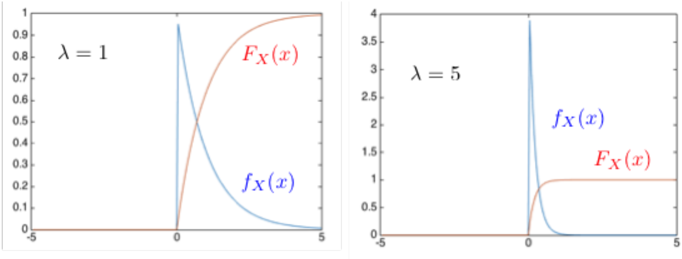

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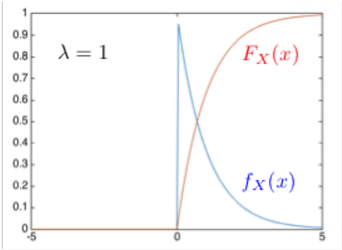

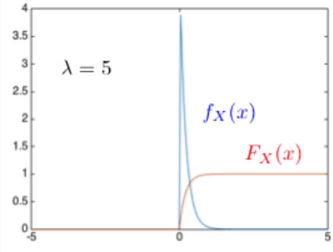

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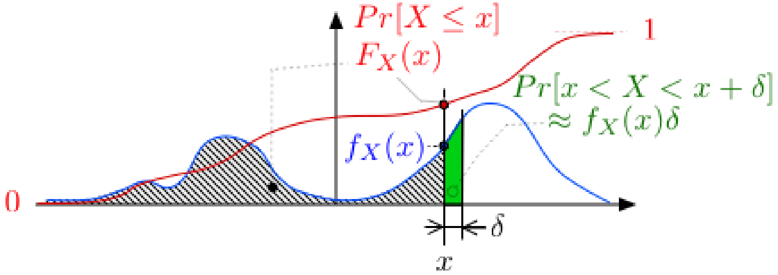
Recall that $Pr[X \in (x, x+\delta)] \approx f_X(x)\delta$.
X "takes" value $n\delta$, for $n \in Z$, with $Pr[X = n\delta] = f_X(n\delta)\delta$

A Picture



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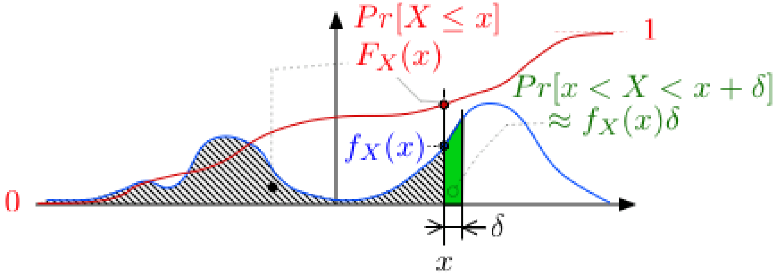

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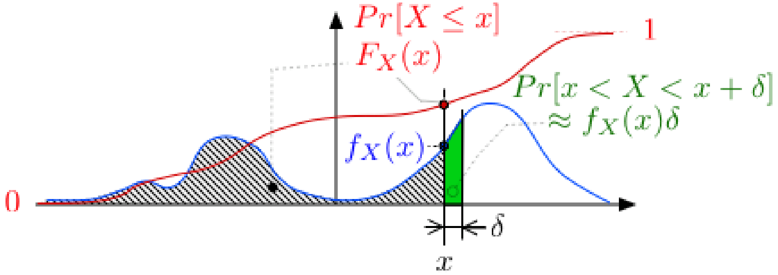

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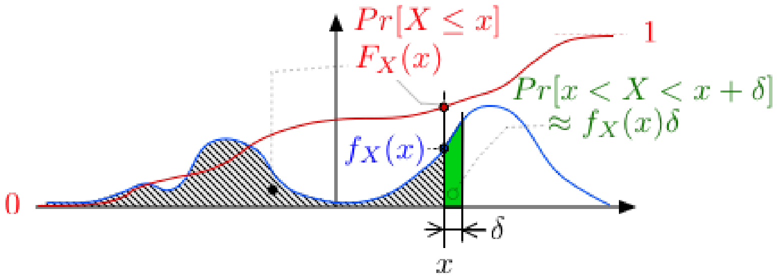

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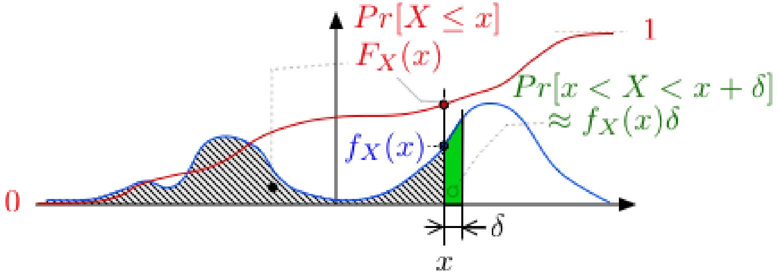

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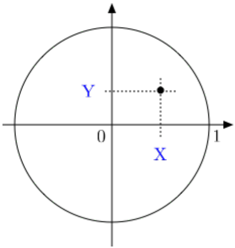

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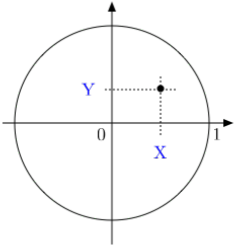

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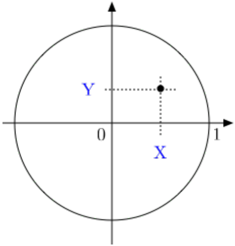

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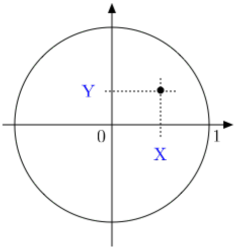

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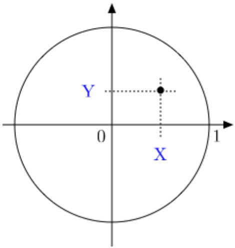

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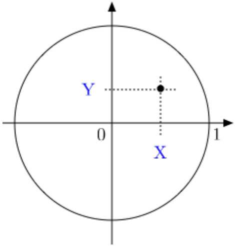

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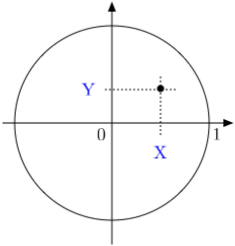

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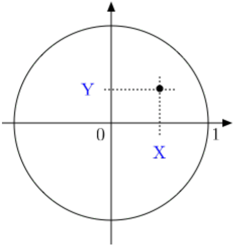

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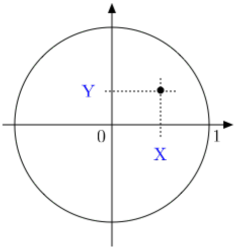

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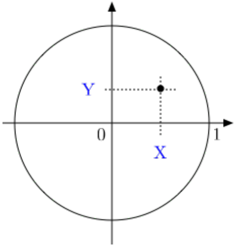

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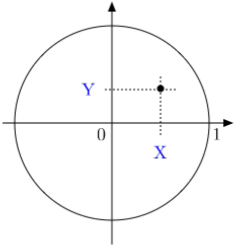

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Proof: As in the discrete case.

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Corollary: For independent random variables, $f_{X|Y}(x,y) = f_X(x)$.

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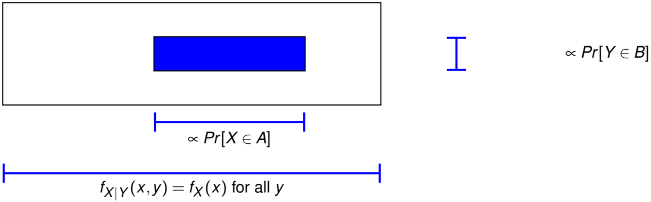

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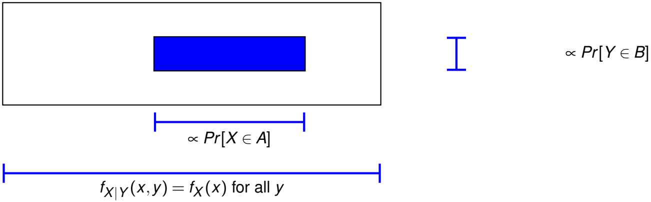

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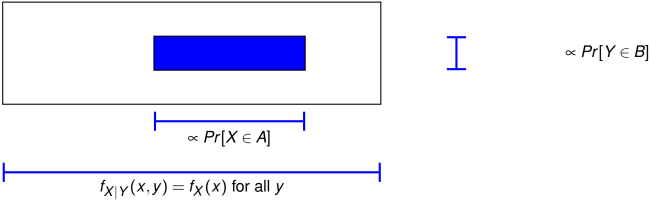

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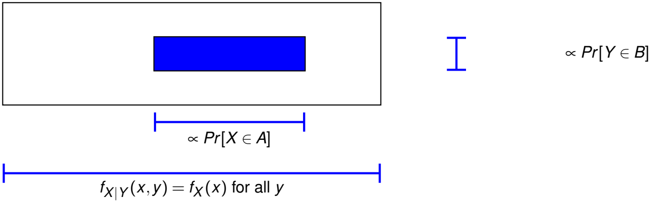

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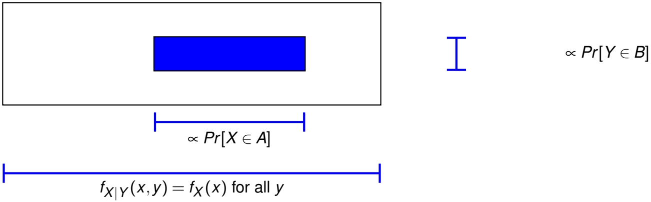

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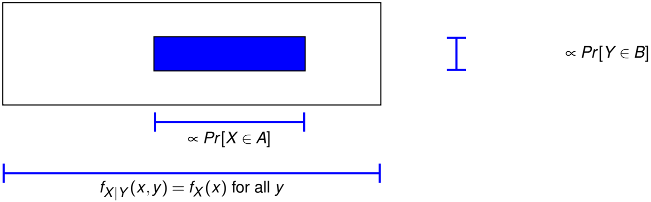

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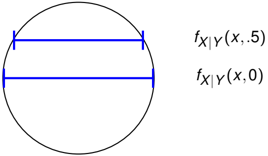

Not independent!

Summary

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3. $U[a,b]$: $f_X(x) = \frac{1}{b-a} 1\{a \leq x \leq b\}; F_X(x) = \frac{x-a}{b-a}$ for $a \leq x \leq b$.
4. $Expo(\lambda)$:
   $f_X(x) = \lambda \exp\{-\lambda x\} 1\{x \geq 0\}; F_X(x) = 1 - \exp\{-\lambda x\}$ for $x \leq 0$.
5. Target:

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5. Target: $f_X(x) = 2x1\{0 \leq x \leq 1\}; F_X(x) = x^2$ for $0 \leq x \leq 1$.
6. Joint pdf: $Pr[X \in (x, x+\delta), Y = (y, y+\delta)) = f_{X,Y}(x,y)\delta^2$.
   6.1 Conditional Distribution: $f_{X|Y}(x,y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$.
   6.2 Independence: $f_{X|Y}(x,y) = f_X(x)$

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