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Integration by Parts: $\int udv = uv - \int vdu$.

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Discrete: Probability of outcome $\rightarrow$ random variables, events.

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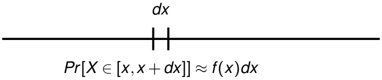

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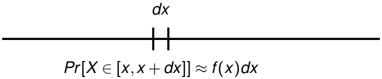

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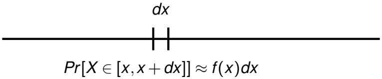

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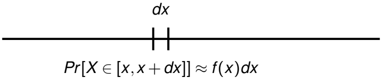

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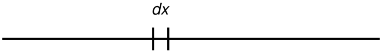

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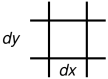

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Sample Space:Ω, $Pr[\omega]$.

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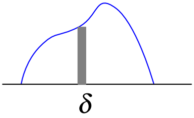
Continuous as Discrete.
$Pr[X \in [x, x+\delta]] \approx f(x)\delta$



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All work for continuous with intervals as events.

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Corollary: For independent random variables, $f_{X|Y}(x,y) = f_X(x)$.

Expo(λ)

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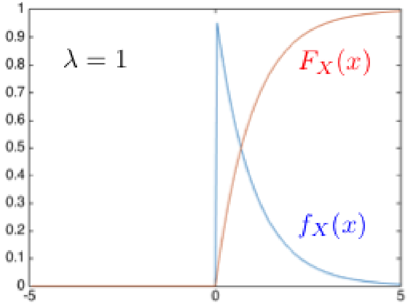

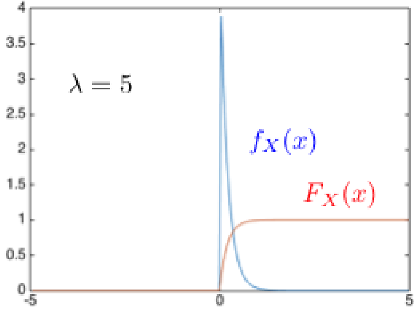

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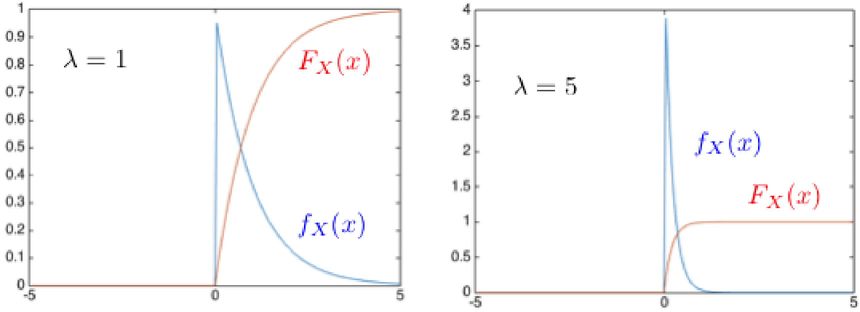

Note that $Pr[X > t] = e^{-\lambda t}$ for $t > 0$.

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Replace b by $b-a$, use $X = U[0,1]$, then $Y = a + (b-a)X$ is $U[a,b]$.

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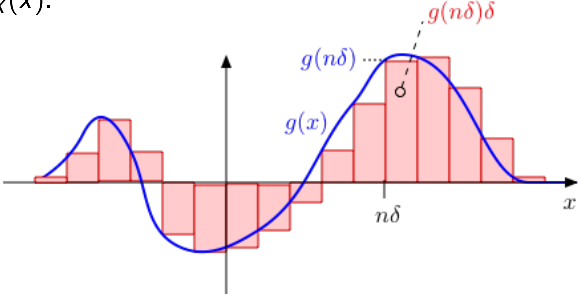

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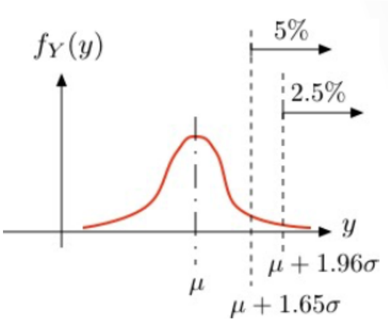

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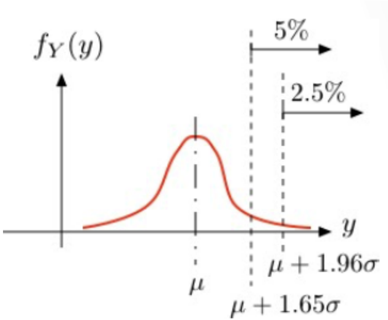

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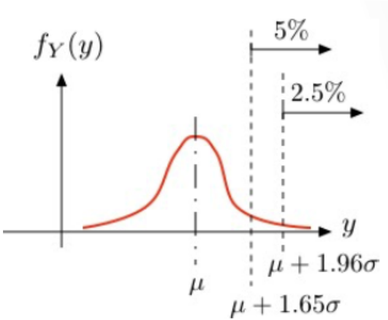

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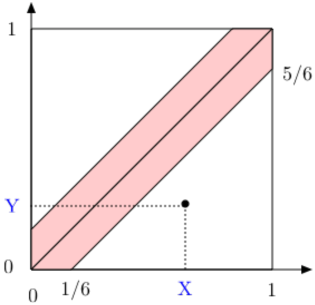

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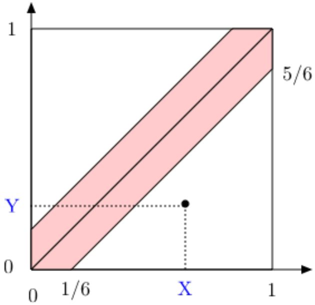

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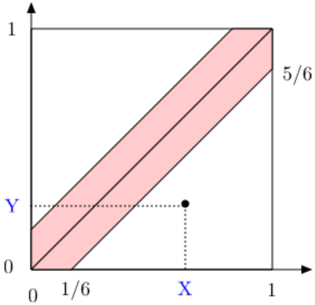

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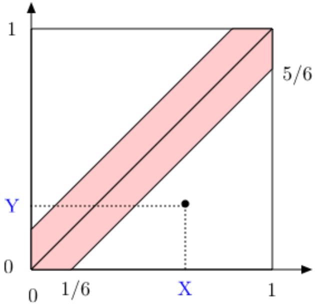

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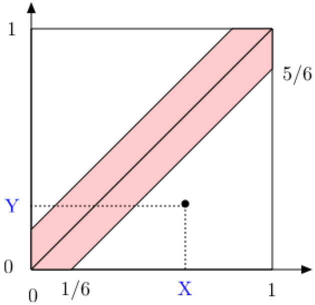

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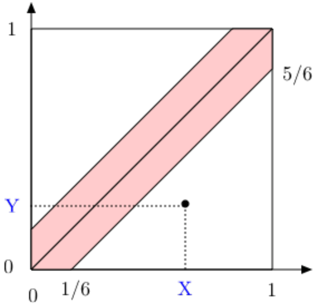

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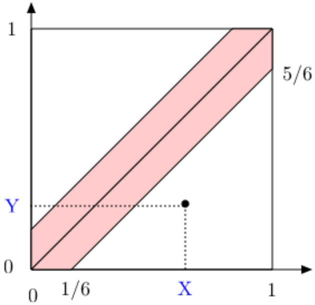

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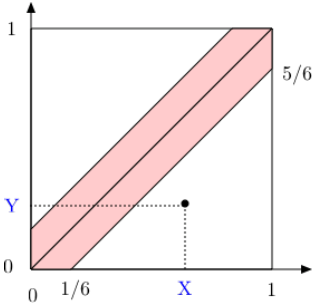

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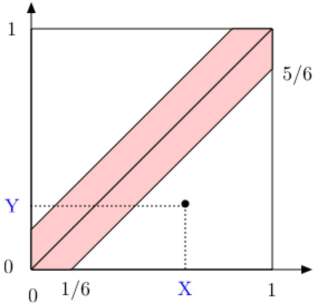

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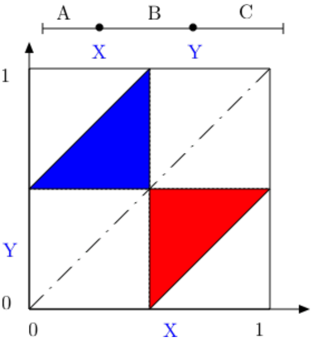

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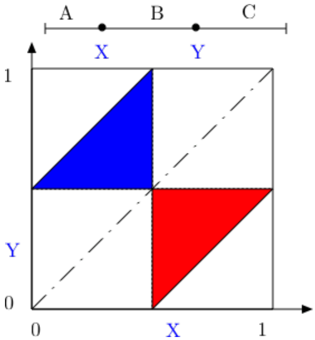

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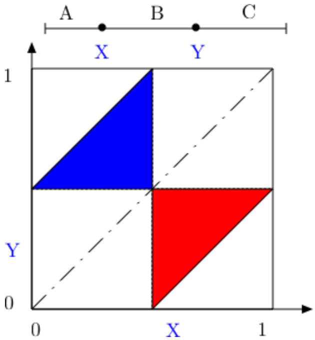

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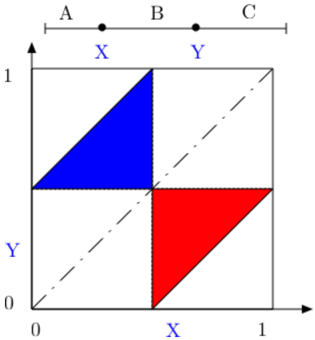

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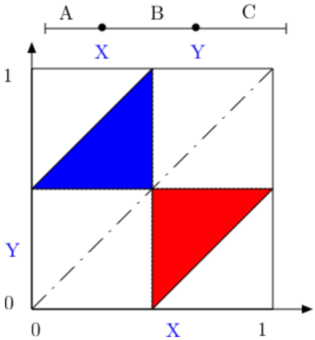

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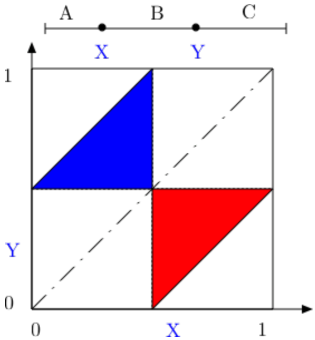

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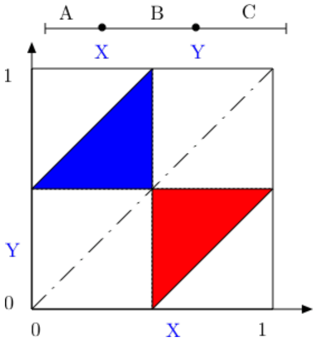

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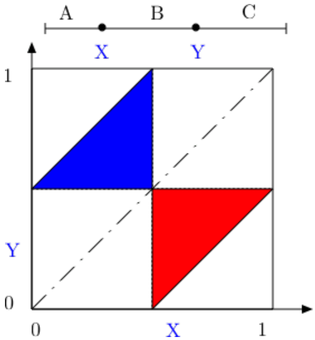

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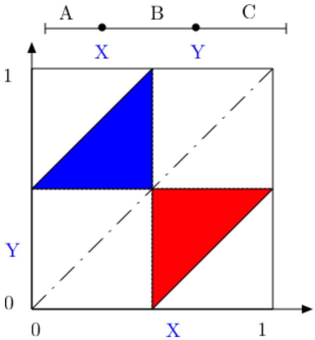

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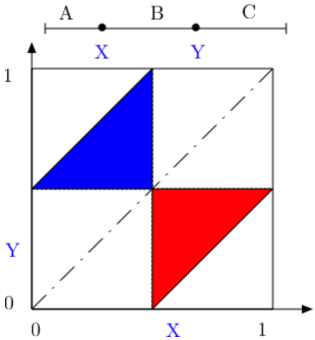

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Thus, $Pr[\text{make triangle}] = 1/4$.

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(C) is an argument for (A), (B) and (D).

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Hence,

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For instance, if $n = 16$, then $SNR(dB) \approx 112dB$.

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