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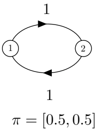

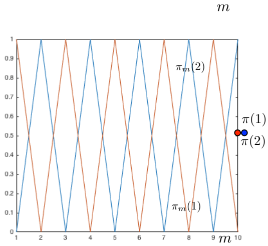

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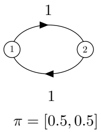

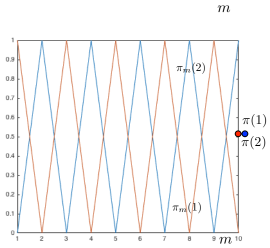

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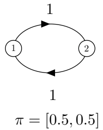

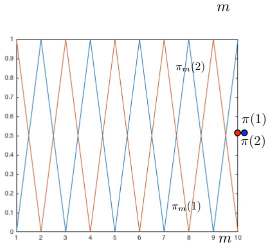

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$\implies$ decompose invariant distribution into d distributions on d colors.

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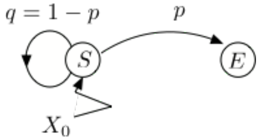

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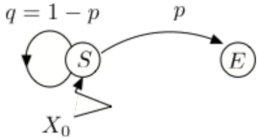

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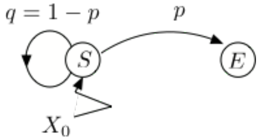

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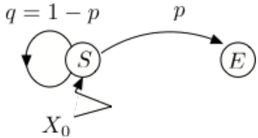

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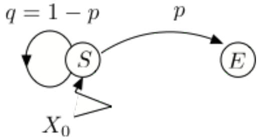

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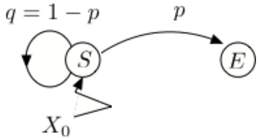

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All are correct. (D) is the "Markov property." Only know where you are.

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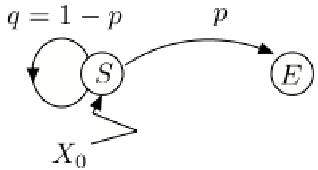

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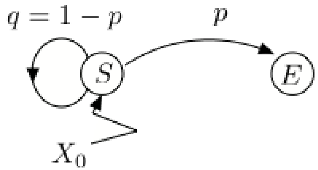

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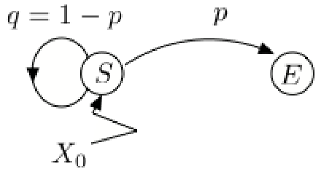

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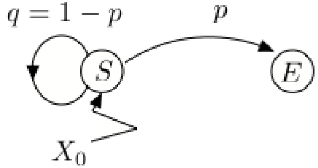

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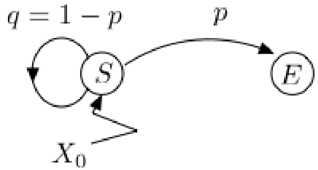

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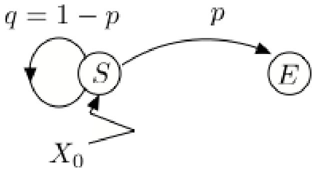

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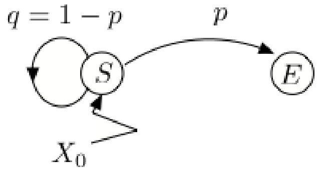

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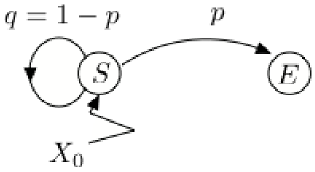

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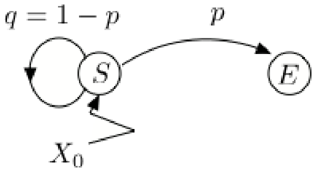

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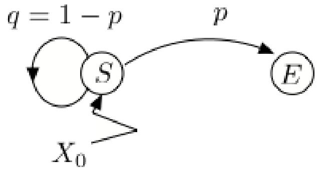

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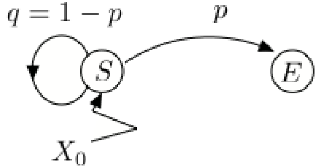

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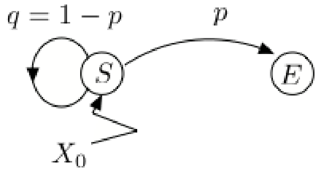

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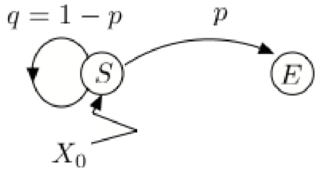

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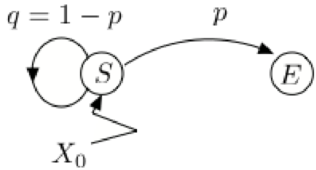

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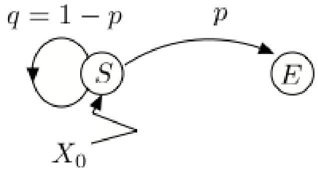

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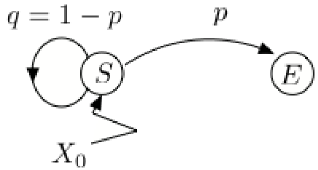

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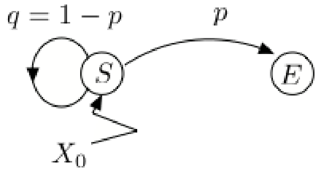

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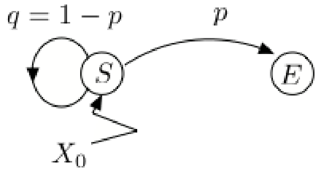

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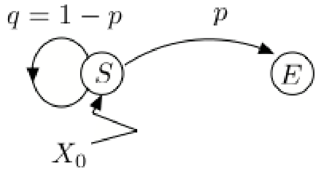

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$$\beta(S) = 1 + q \times \beta(S) + p \times 0.$$

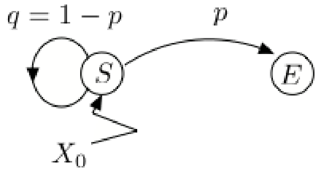
Justification: N – number of steps until E, starting from S.
N' – number of steps until E, after the second visit to S.
And $Z = 1\{\text{first flip } = H\}$. Then,

$$N = 1 + (1 - Z) \times N' + Z \times 0.$$

Z and N' are "independent." $E[N'] = E[N] = \beta(S)$.
Hence, taking expectation,

First Passage Time - Example 1

Let's flip a coin with $Pr[H] = p$ until we get H. How many flips in expectation?



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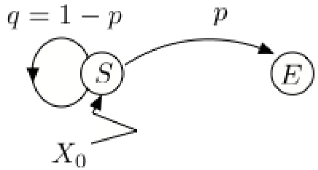

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- $X_n = T$, if last flip was T and we are not done

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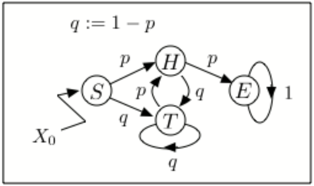

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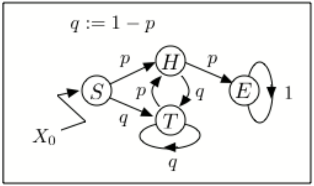

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Which one is correct?
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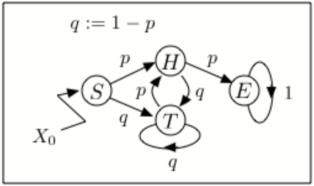

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(A) Expected time from S to E.
$\beta(S) = Pr[H]E[\beta(S)|H] + Pr[T]E[\beta(S)|T]$

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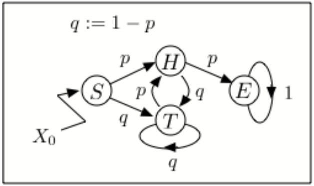

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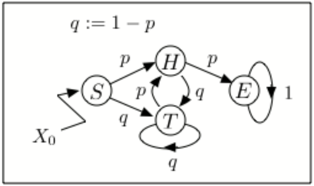

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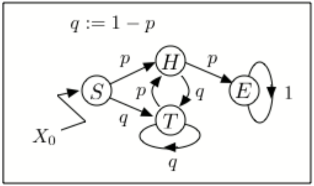

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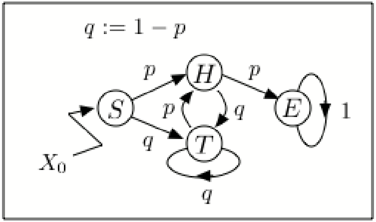

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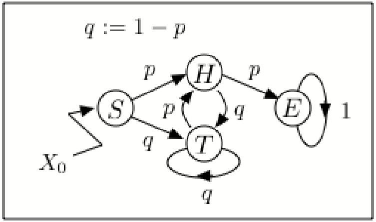

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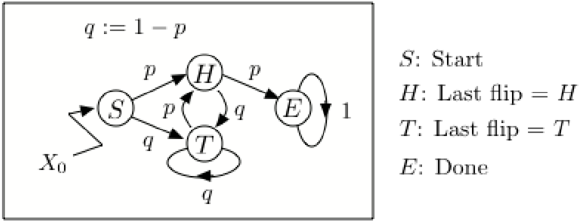

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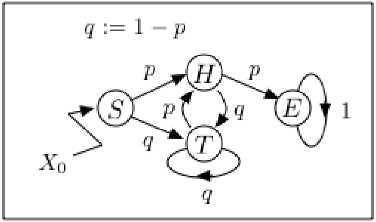

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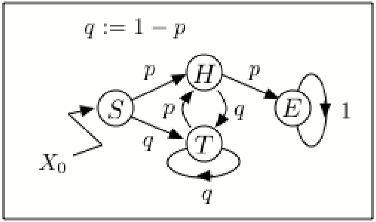

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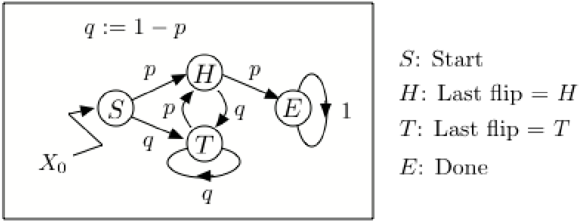

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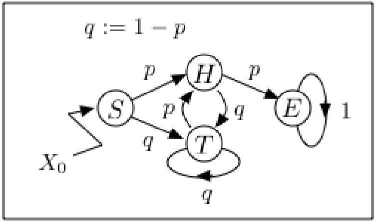

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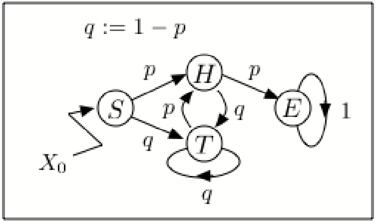

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$$\begin{aligned}\beta(S) &= 1 + p \times \beta(H) + q \times \beta(T)\\ \beta(H) &= 1 + p \times 0 + q \times \beta(T)\\ \beta(T) &= 1 + p \times \beta(H) + q \times \beta(T).\end{aligned}$$

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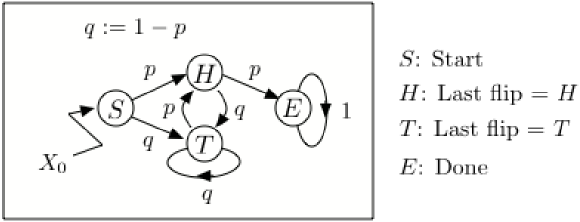

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Solving, we find $\beta(S) = 2 + 3qp^{-1} + q^2 p^{-2}$.

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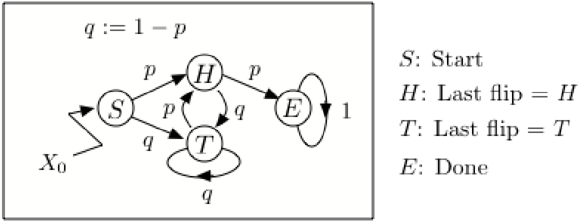

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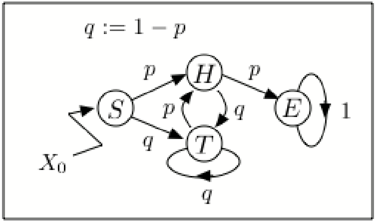

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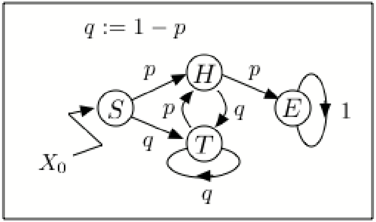

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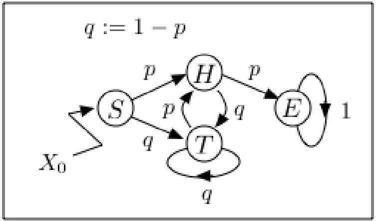
Let us justify the first step equation for $\beta(T)$.

Hitting Time - Example 2



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Let us justify the first step equation for $\beta(T)$. The others are similar.

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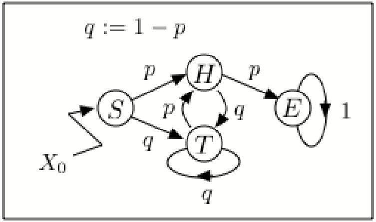

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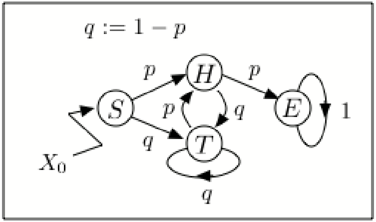
Let us justify the first step equation for $\beta(T)$. The others are similar.

$N(T)$ – number of steps, starting from T until the MC hits E.

Hitting Time - Example 2



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Let us justify the first step equation for $\beta(T)$. The others are similar.

$N(T)$ – number of steps, starting from T until the MC hits E.
$N(H)$ – be defined similarly.

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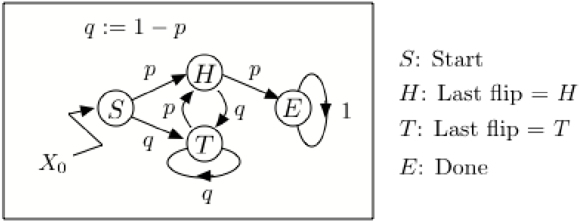

Let us justify the first step equation for $\beta(T)$. The others are similar.

$N(T)$ – number of steps, starting from T until the MC hits E.
$N(H)$ – be defined similarly.
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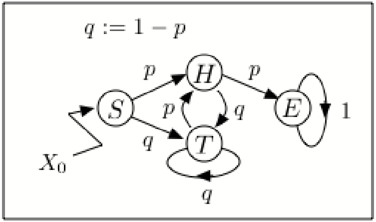

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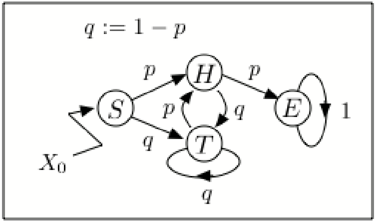
$$N(T) = 1 + Z \times N(H) + (1 - Z) \times N'(T)$$

Hitting Time - Example 2



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Let us justify the first step equation for $\beta(T)$. The others are similar.

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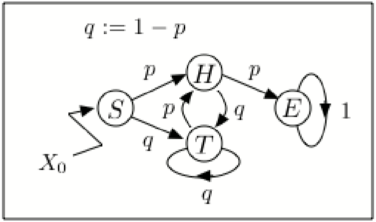

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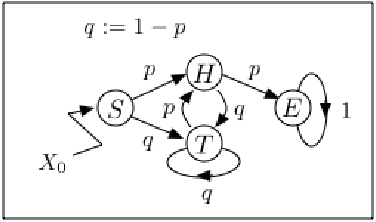

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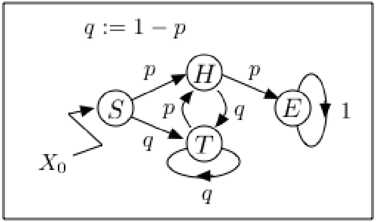

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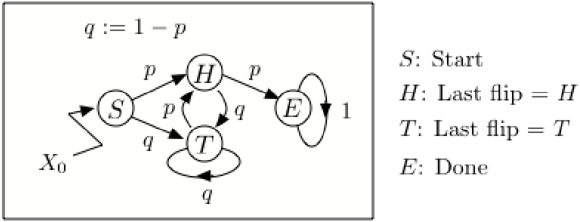

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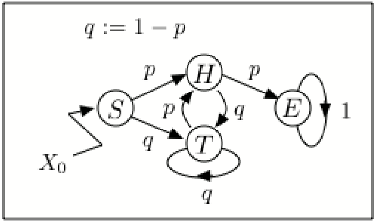

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i.e.,

$$\beta(T) = 1 + p\beta(H) + q\beta(T).$$

Hitting Time - Example 3

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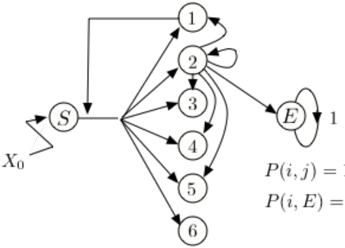

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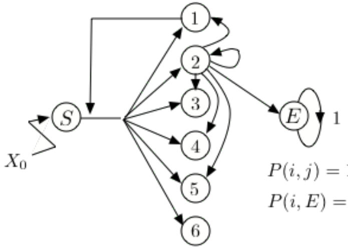

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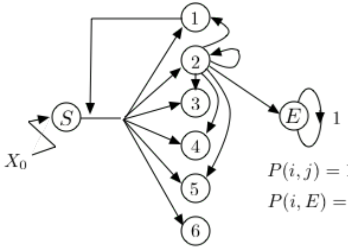

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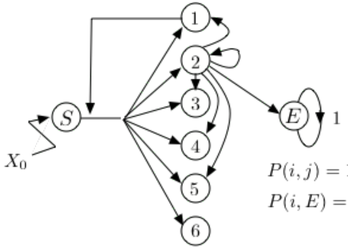

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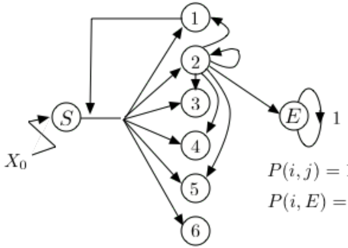

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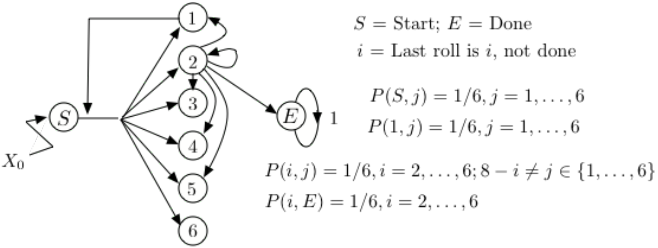

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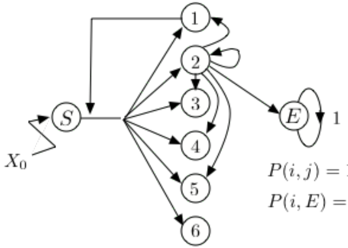

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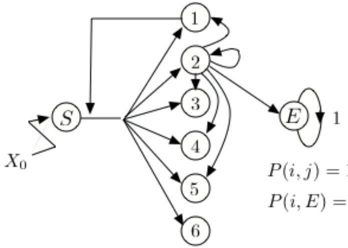

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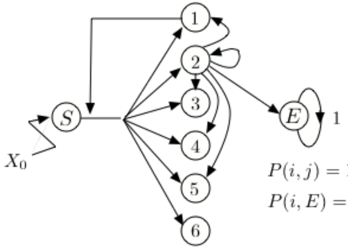

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$$\Rightarrow \cdots \beta(S) = 8.4.$$

Here before There - A before B

Game of “heads or tails” using coin with ‘heads’ probability $p < 0.5$.

Here before There - A before B

Game of “heads or tails” using coin with ‘heads’ probability $p < 0.5$.
Start with \$10.

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Start with \$10.

Each step, flip yields ‘heads’, earn \$1.

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Start with \$10.
Each step, flip yields ‘heads’, earn \$1. Otherwise, lose \$1.

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What is the probability that you reach \$100 before \$0?

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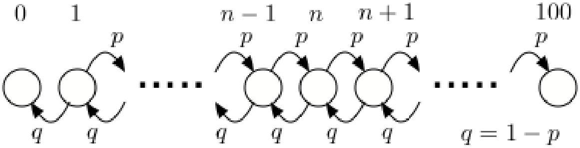

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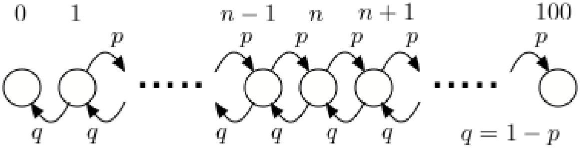

Let $\alpha(n) = Pr[\text{reach 100 before } 0 | \text{at } n]$ for $n = 0, 1, \ldots, 100$.

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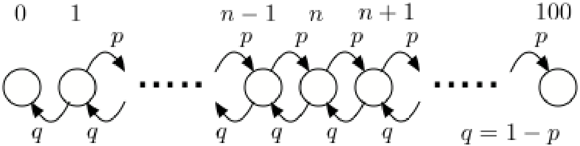

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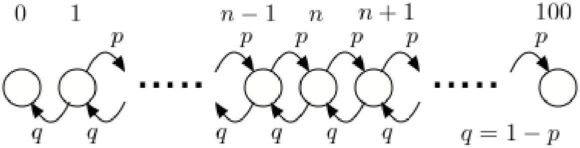

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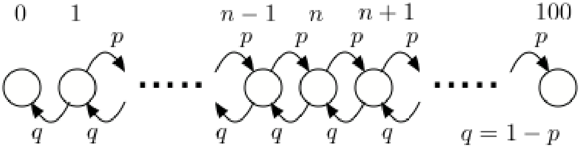

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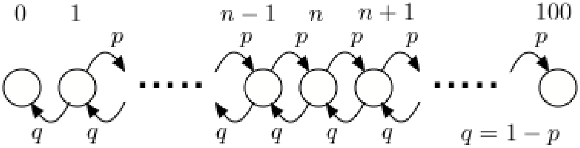

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(B) is incorrect, 0 is bad.
(D) is incorrect. Confuses expected hitting time with *A* before *B*.

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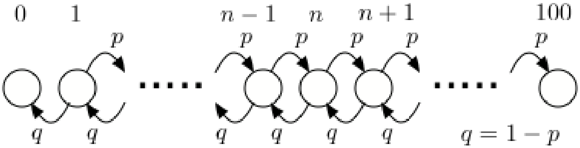

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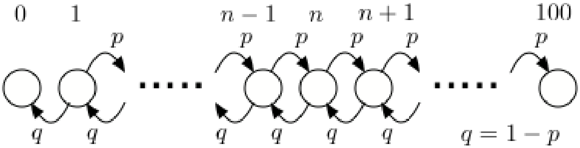

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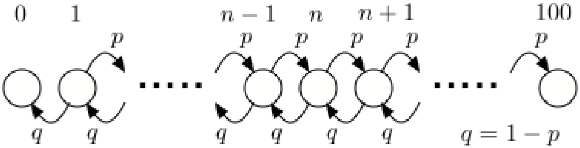

Let $\alpha(n) = Pr[\text{reach 100 before } 0 | \text{at } n]$ for $n = 0, 1, \ldots, 100$.

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Here before There - A before B

Game of "heads or tails" using coin with 'heads' probability $p < 0.5$.
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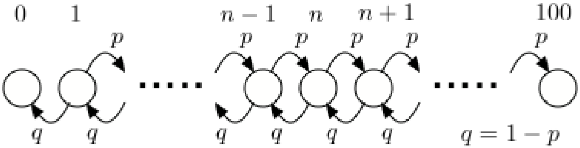

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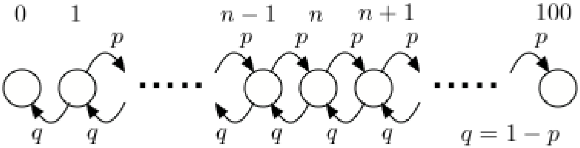

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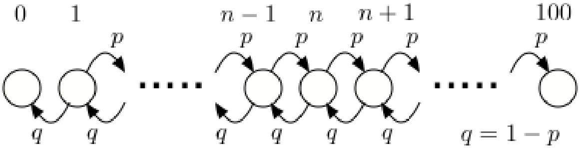

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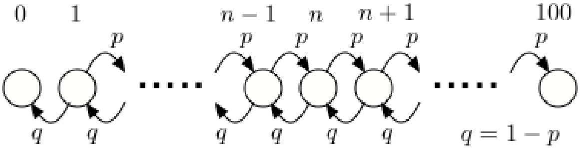

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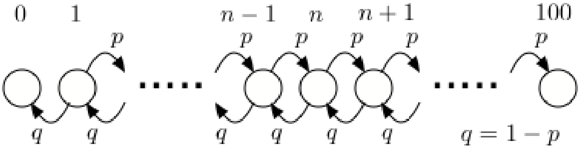

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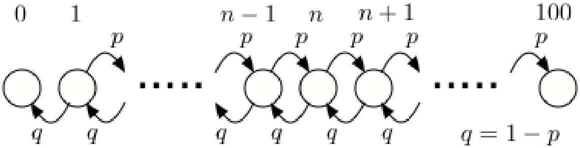

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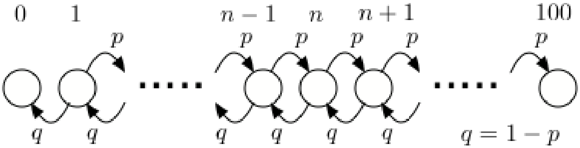

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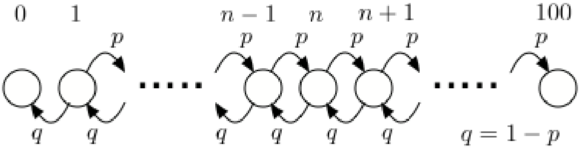

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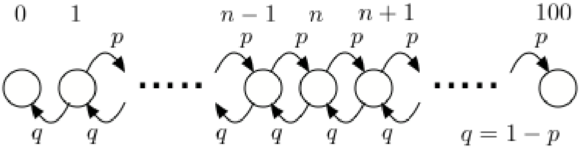

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Here before There - A before B

Game of “heads or tails” using coin with ‘heads’ probability $p = .48$.

Here before There - A before B

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Here before There - A before B

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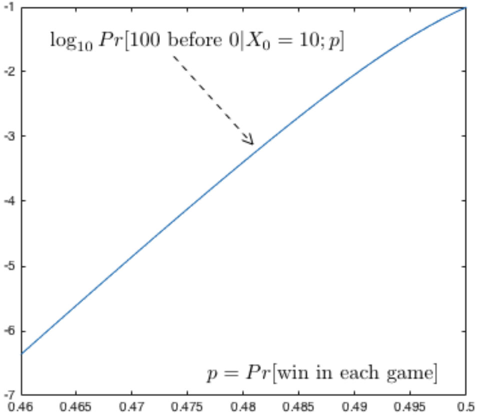

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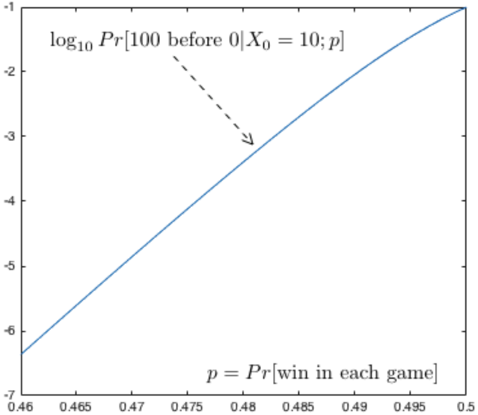

Less than 1 in a 1000.

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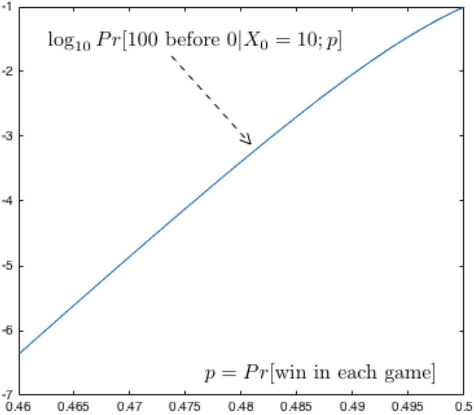

Less than 1 in a 1000. Moral of example:

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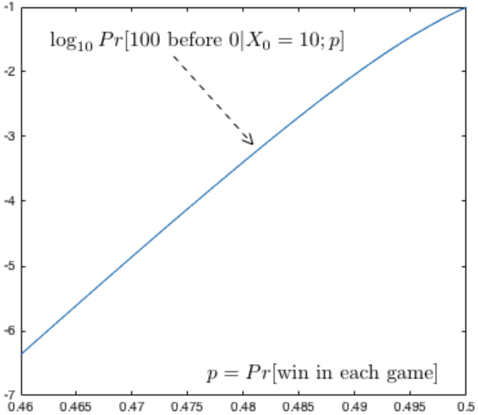

Less than 1 in a 1000. Moral of example: Money in Vegas

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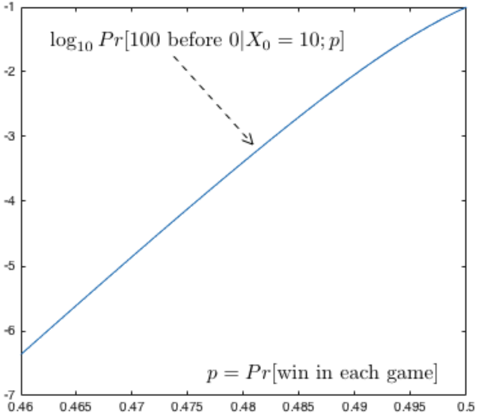

Less than 1 in a 1000. Moral of example: Money in Vegas stays in Vegas.

First Step Equations

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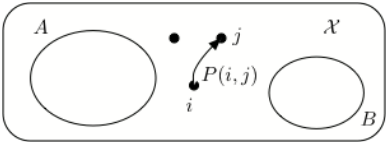

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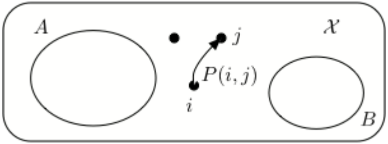

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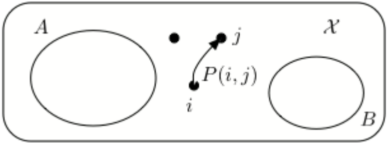

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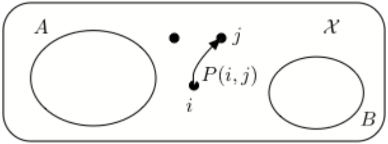

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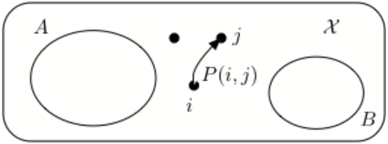

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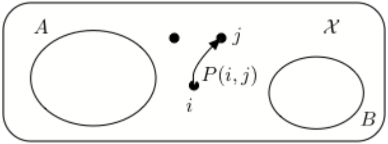

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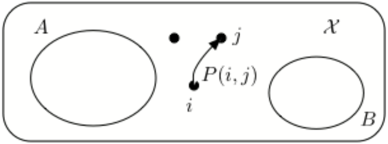

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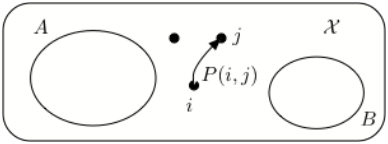

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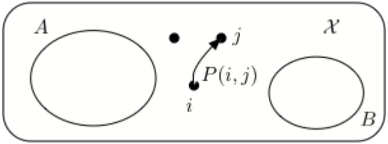

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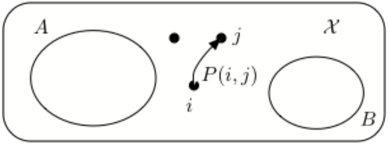

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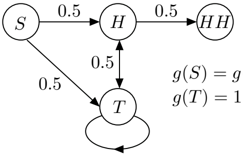

$g(S) = g(H) = g(HH) = 0$
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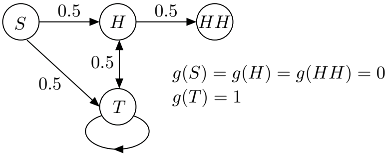

FSE:

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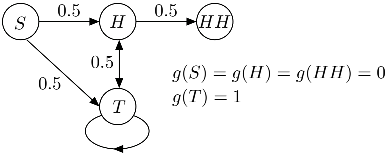

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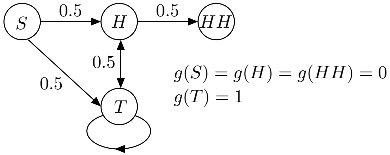

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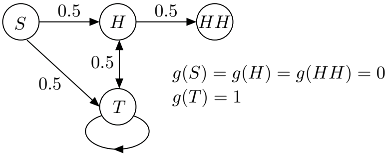

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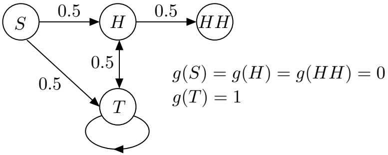
Example

Flip a fair coin until you get two consecutive Hs.

What is the expected number of Ts that you see?



FSE:

$$
\begin{aligned}
\gamma(S) &= 0 + 0.5\gamma(H) + 0.5\gamma(T) \\
\gamma(H) &= 0 + 0.5\gamma(HH) + 0.5\gamma(T) \\
\gamma(T) &= 1 + 0.5\gamma(H) + 0.5\gamma(T) \\
\gamma(HH) &= 0.
\end{aligned}
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Solving, we find $\gamma(S) = 2.5$.

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CS70: Lecture 27

1. Continuous Probability
2. Normal Distribution
3. Central Limit Theorem
4. Confidence Intervals
5. Wrapup.

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6. Variance of Sum of Independent RVs: If X_n are pairwise independent, $var[X_1 + \cdots + X_n] = var[X_1] + \cdots + var[X_n]$
7. Joint Density function:
   $Pr[X \in [x, x+\delta], Y \in [y, y+\delta]] \approx f_{X,Y}(x,y)\delta^2$.
8. Conditional Density:
   $Pr[X \in [x, x+\delta] | Y = y] \approx f_{X|Y}(x,y)dx = \frac{f_{X,Y}(x,y)}{f_Y(y)}dx$.

Summary

1. Bayes' Rule: Replace $\{X = x\}$ by $\{X \in (x, x + \varepsilon)\}$.
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Random Thoughts

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  E.g., facebook friends effect, ignoring inconvenient articles.

Confirmation Bias

Confirmation bias: tendency to search for, interpret, and recall information in a way that confirms one's beliefs or hypotheses, while giving less consideration to alternative possibilities.

Confirmation biases contribute to overconfidence in personal beliefs and can maintain or strengthen beliefs in the face of contrary evidence.

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  E.g., valuing confirming versus contrary evidence.
- Biased memory.
  E.g., remember facts that confirm beliefs and forget others.

Confirmation Bias: An experiment

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There are two bags.

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There are two bags.

One with 60% red balls and 40% blue balls;

Confirmation Bias: An experiment

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One selects one of the two bags.

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As one draws balls one at time,

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As one draws balls one at time, one asks people to declare whether they think one draws from the first or second bag.

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One with 60% red balls and 40% blue balls;
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One selects one of the two bags.

As one draws balls one at time, one asks people to declare whether they think one draws from the first or second bag.

Surprisingly, people tend to be reinforced in their original belief, even when the evidence accumulates against it.

Report Data not Opinion!

A bag with 60% red, 40% blue or vice versa.

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Each person pulls ball, reports opinion on which bag:

Report Data not Opinion!

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Says "majority blue" or "majority red."

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Does not say what color their ball is.

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What happens if first two get blue balls?

Third hears two say blue, so says blue, whatever she sees.

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Everyone says blue...

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Problem: Each person reported honest opinion

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Everyone says blue...forever ...and ever.

Problem: Each person reported honest opinion rather than data!

Being Rational: 'Thinking, Fast and Slow'

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In this book, Daniel Kahneman discusses examples of our irrationality.

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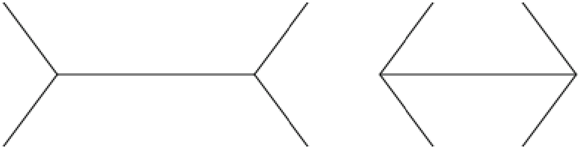
- Perception: Which horizontal line is longer?



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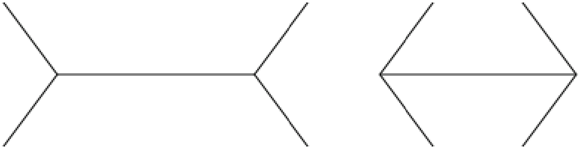

It is difficult to think clearly!

Really?

Judges at Lousiana give longer sentences when LSU gives upset losses.

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Judges give larger sentences when hungry.

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Certainty is the enemy?
Unless you work hard!

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Certainty is the enemy?
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Uh oh.

Certainty is the enemy?
Unless you work hard! You have the internet.
You have your intellect.
...and (most important) your integrity.

What to Remember?

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Professor, what should I remember about probability from this course?

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I mean, after the final.

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Parting Thoughts

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You have learned a lot in this course!

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Proofs,

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Proofs, Graphs,

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Proofs, Graphs, Mod(p),

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how to handle stress,

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Proofs, Graphs, Mod(p), RSA, Reed-Solomon, Decidability, Probability, ... ,
how to handle stress, how to sleep less,

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how to handle stress, how to sleep less, how to keep smiling, ...

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  Useful?

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Useful? You bet!

Arguments, reasoning.

What you know: slope, limit.

Arguments, reasoning.

What you know: slope, limit.
Plus: definition.

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  Plus: definition.
yields calculus.

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Knowing how to program

Arguments, reasoning.

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  Plus: definition.
yields calculus.
  Minimization, optimization, …..

Knowing how to program plus some syntax (google) gives the ability to program.

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Arguments, reasoning.

What you know: slope, limit.
  Plus: definition.
yields calculus.
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Knowing how to reason plus some definition

Arguments, reasoning.

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Knowing how to reason plus some definition gives calculus.

Arguments, reasoning.

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Discrete Math: basics are counting, how many, when are two sets the same size?

Arguments, reasoning.

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Discrete Math: basics are counting, how many, when are two sets the same size?

Probability:

Arguments, reasoning.

What you know: slope, limit.
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Discrete Math: basics are counting, how many, when are two sets the same size?

Probability: division.

Arguments, reasoning.

What you know: slope, limit.
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Knowing how to program plus some syntax (google) gives the ability to program.

Knowing how to reason plus some definition gives calculus.

Discrete Math: basics are counting, how many, when are two sets the same size?

Probability: division.

...plus reasoning.

What's Next?

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What’s Next?

Professor, I loved this course so much!

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- EE126: Probability in EECS: An Application-Driven Course:

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  Hands on labs with python experiments (GPS, Shazam, ...).

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- EE121: Digital Communication:

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- EE229A: Information Theory; EE229B: Coding Theory.

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Final Thoughts

Final Thoughts

More precisely:

Final Thoughts

More precisely: Some thoughts about the final ....

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How to study for the final?

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides;

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes;

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes; Discussion Problems;

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes; Discussion Problems; HW

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes; Discussion Problems; HW
- Approximate Coverage: Probability 2/3, Discrete Math: 1/3.

Final Thoughts

More precisely: Some thoughts about the final ....

How to study for the final?

- Lecture Slides; Notes; Discussion Problems; HW
- Approximate Coverage: Probability 2/3, Discrete Math: 1/3.
- Every question topic covered in at least two places. Most will be covered in all places.

Finally....

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Thanks for taking the course!

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Thanks to the CS70 Staff:

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Thanks to the CS70 Staff:

- The Terrific Tutors
- The Rigorous Readers
- The Thrilling TAs
- The Amazing Assistants

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Good studying!!!