Climb an infinite ladder?

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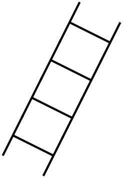
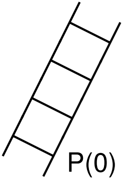


Climb an infinite ladder?

$P(0)$



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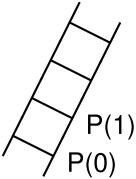
$$P(0)$$
$$\forall k, P(k) \implies P(k+1)$$



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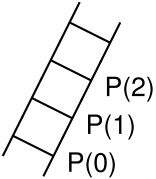

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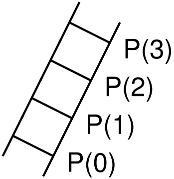

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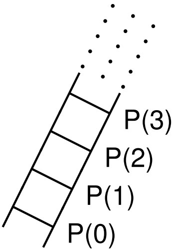

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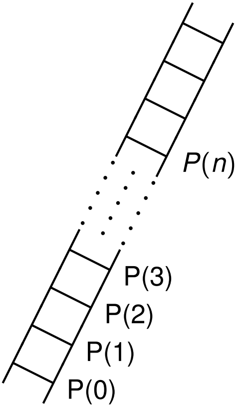

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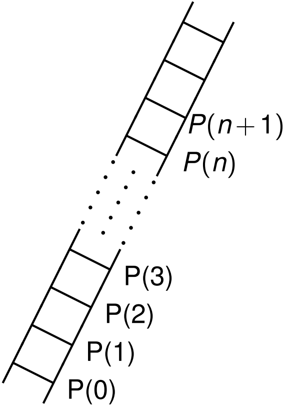

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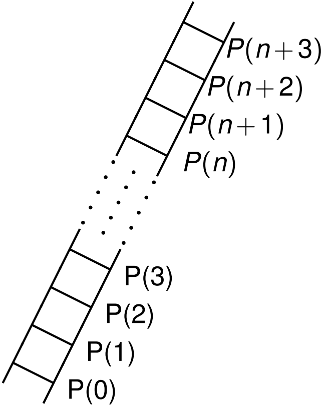

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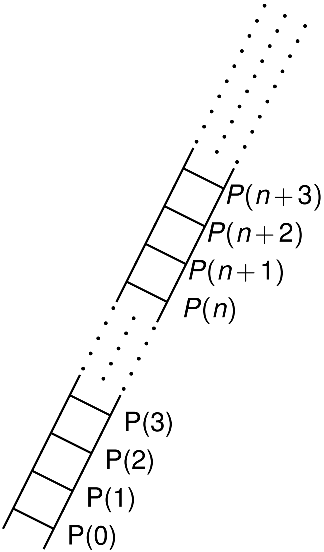

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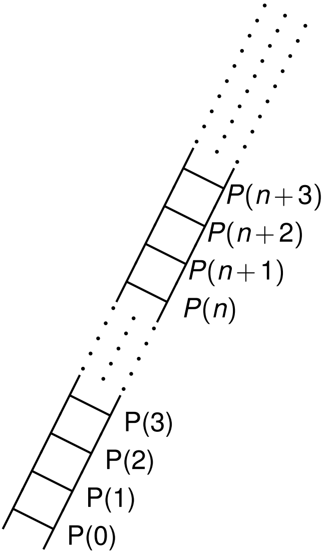

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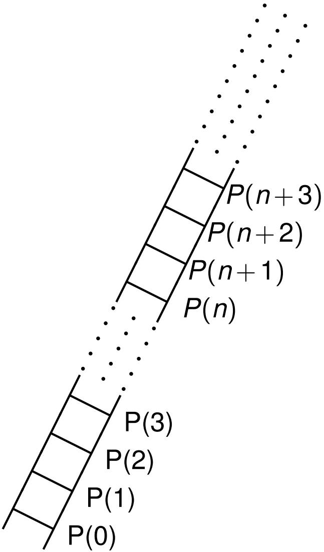

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Your favorite example of forever..or the natural numbers...

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Thm: If there are n villagers with green eyes they leave on day n.

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Wait! Visitor added no information.

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If 66 villagers out of the 100 had green eyes, what would happen?

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Strong Induction and Recursion.

Thm: For every natural number $n \geq 12$, $n = 4x + 5y$.

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Base cases: P(12) , P(13) , P(14) , P(15). Yes.

Strong Induction and Recursion.

Thm: For every natural number $n \geq 12$, $n = 4x + 5y$.

Instead of proof, let's write some code!

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Slight differences: showed for all $n \geq 16$ that $\wedge_{i=4}^{n-1} P(i) \implies P(n)$.

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The induction principle works on the natural numbers.

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In some sense, the natural numbers.

Stable Matching Problem

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- n candidates and n jobs.

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- Minimize difference between preference ranks.

The best laid plans..

Consider the pairs..

- Cal Bears and the Pac-12
- Wake Forest and the ACC

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Cal Bears prefers the ACC

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The ACC prefers Cal Bears.

Uh..oh.

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Uh..oh. Sad Pac-12, (and Wake Forest.)

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Produce a matching where there are no

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Example: A matching $S = \{(CalBears, Pac12); (WakeForest, ACC)\}$.

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Not a great example of stable matching, but interesting exercise in "selfish" incentives.

A stable matching??

Given a set of preferences.

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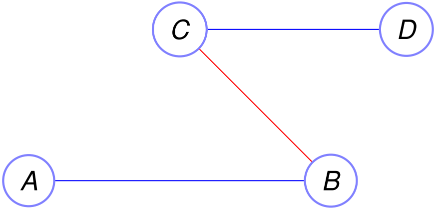
Consider a single type version: stable roommates.

A	B	C	D
B	C	A	D
C	A	B	D
D	A	B	C



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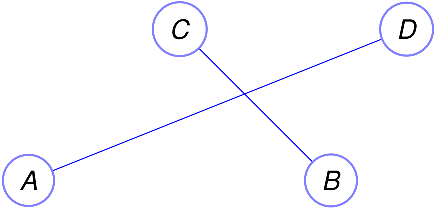

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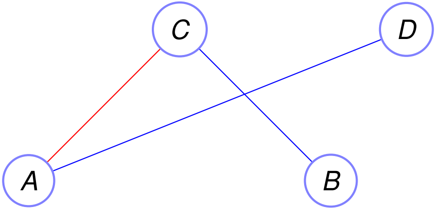

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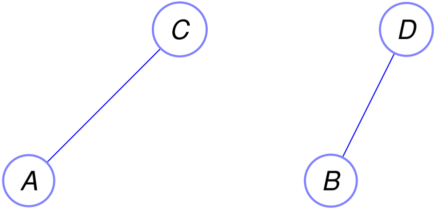

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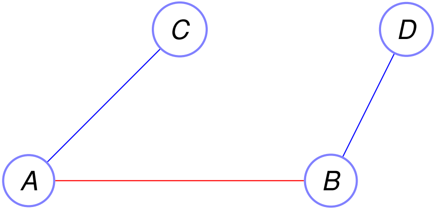

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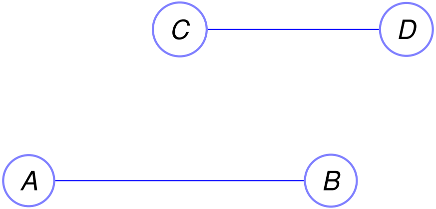

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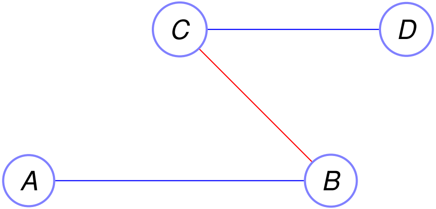

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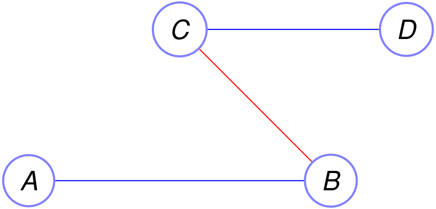

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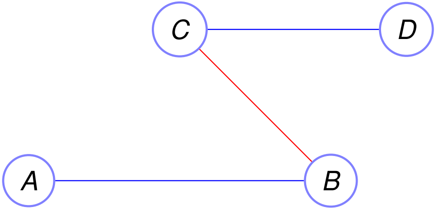

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Stop when each job gets exactly one proposal.

Example.

Jobs			
A	1	2	3
B	1	2	3
C	2	1	3

Candidates			
1	C	A	B
2	A	B	C
3	A	C	B

Example.

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Candidates			
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	Day 1	Day 2	Day 3	Day 4	Day 5
1					
2					
3					

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	Day 1	Day 2	Day 3	Day 4	Day 5
1	A, B				
2	C				
3					

Example.

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A	1	2	3
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	Day 1	Day 2	Day 3	Day 4	Day 5
1	A, ~~B~~				
2	C				
3					

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	Day 1	Day 2	Day 3	Day 4	Day 5
1	A, ~~B~~	A			
2	C	B, C			
3					

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1	A, ~~B~~	A	A , C		
2	C	B, ~~C~~	B		
3					

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3					B

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	Day 1	Day 2	Day 3	Day 4	Day 5
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3					B

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2. Each candidate rejects all but their favorite proposer (whom they put on a string.)
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Stop when each job gets exactly one proposal.
Does this terminate?

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Is this “better” for jobs or candidates?

The Propose and Reject Algorithm.

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What does “better” even mean?

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Total size of lists? *n* jobs, *n* length list.

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Terminates in $\leq n^2$ steps!

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Poll

Question: It just gets better for candidates,

(A) By induction on days.

(B) When the economy is good.

(C) The candidate can always keep the job on the string.

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Economics: Study of choice. Freedom of choice.

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Lemma: Every job is matched at end.

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(A) Implies: no unmatched job at end.

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(C) From Algorithm: unmatched job would ask everyone.

Poll: The argument for termination ...

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(D) is false.

Matching is Stable.

Lemma: There is no rogue couple for the matching formed by traditional marriage algorithm.

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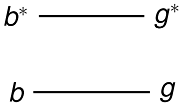

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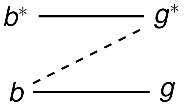

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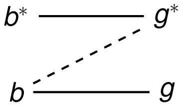

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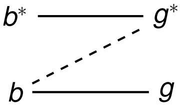

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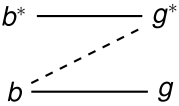

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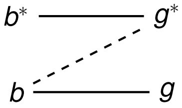

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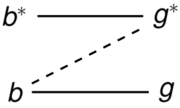

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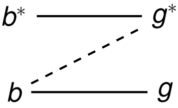

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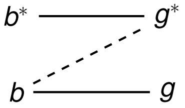

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Poll

Proof of Job Propose and Reject produces stable pairing uses?

(A) Contradiction.

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Poll

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(A), (B), (C), (E).(Maybe (D) internally. Semantics.)

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Is the Job-Proposes better for jobs?

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Consider pairing: $(A, 1), (B, 2)$.

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Pairing *T*: $(A,2),(B,1)$.

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Which is optimal for A?

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Which is optimal for *A*? *S* Which is optimal for *B*? *S*

Understanding Optimality: by example.

A:	1,2	1:	A,B
B:	1,2	2:	B,A

Consider pairing: $(A,1),(B,2)$.

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Notice: only one stable pairing.
If (A,2) are pair, (A,1) is rogue couple.
So this is the best B can do in a stable pairing.
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Pessimality?

Job Propose and Candidate Reject is optimal!

For jobs?

Job Propose and Candidate Reject is optimal!

For jobs? For candidates?

Job Propose and Candidate Reject is optimal!

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Theorem: Job Propose and Reject produces a job-optimal pairing.

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Proof:

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Rogue couple for S.

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Used Well-Ordering principle...

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Used Well-Ordering principle...Induction.

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What did proof use?

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(A) Algorithm.

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(C) *First* job b, rejected by optimal candidate g

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There exists a better stable S.

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Theorem: Job Propose and Reject produces candidate-pessimal pairing.

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Structural statement: Job optimality

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Structural statement: Job optimality $\implies$ Candidate pessimality.

Quick Questions.

How does one make it better for candidates?

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Propose and Reject - stable matching algorithm:

Quick Questions.

How does one make it better for candidates?

Propose and Reject - stable matching algorithm:
One side proposes.

Quick Questions.

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Jobs Propose $\implies$ job optimal.

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Candidates propose.

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Residency Matching..

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The method was used to match residents to hospitals.

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Hospital optimal....

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..until 1990's...

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Another variation: couples.

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Takeaways.

Analysis of cool algorithm with interesting goal: stability.

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"Economic": different utilities.

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Proofs carefully use definition:
 Stability:
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 Optimality proof:
  Job Optimality:
   contradiction of the existence of a better *stable* pairing.
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Life Lesson:ask,

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"Economic": different utilities.

Definition of optimality: best utility in stable world.

Action gives better results for individuals but gives instability.

Induction over steps of algorithm.

Proofs carefully use definition:
  Stability:
    Improvement Lemma plus every day the job gets to choose.
  Optimality proof:
    Job Optimality:
      contradiction of the existence of a better *stable* pairing.
        that is, no rogue couple by improvement, job choice,
          and well ordering principle.
    Candidate Pessimality:
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Life Lesson: ask, you will do better

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