Lecture 5: Graphs.

Graphs!

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Graphs!

Definitions: model.

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Graphs!

- Definitions: model.
- Fact!

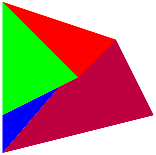
Lecture 5: Graphs.

Graphs!
  Definitions: model.
  Fact!

Lecture 5: Graphs.

Graphs!
  Definitions: model.
  Fact!

Map Coloring.



Map Coloring.

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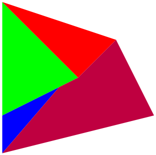
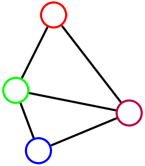
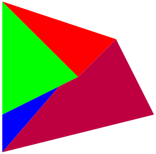
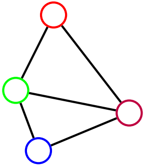




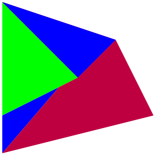
Map Coloring.

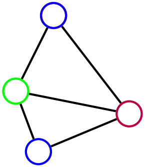




Fewer Colors?

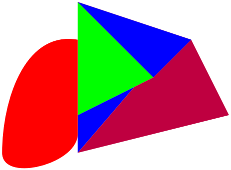
Map Coloring.

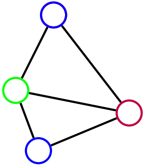




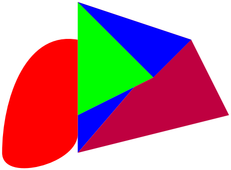
Yes! Three colors.

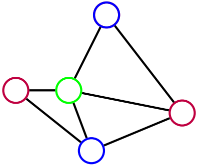
Map Coloring.



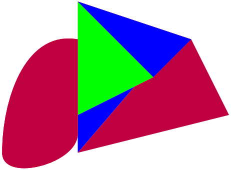


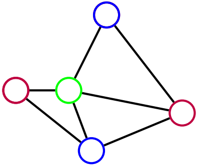
Map Coloring.





Map Coloring.





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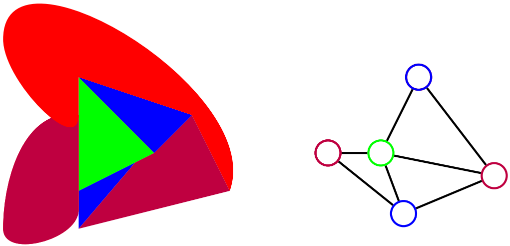

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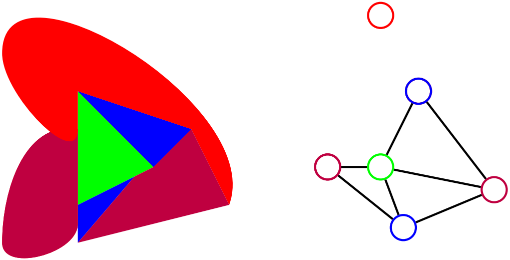

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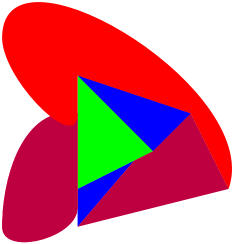

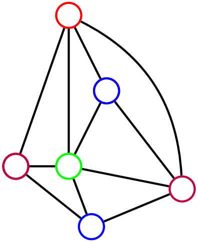

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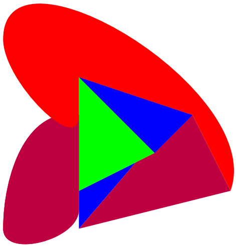
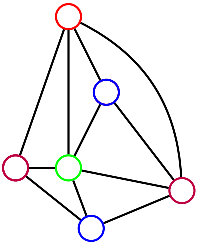
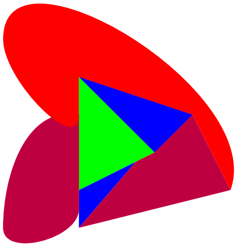
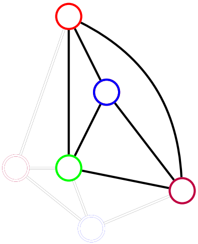




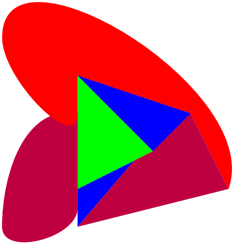
Fewer Colors?

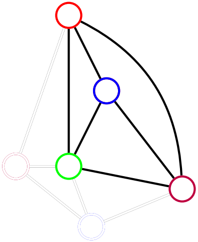
Map Coloring.





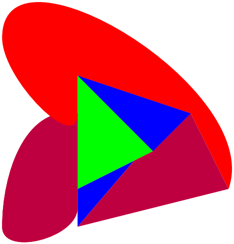
Map Coloring.

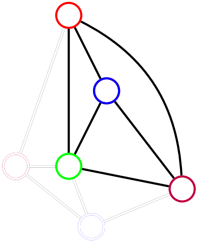




Four colors required!

Map Coloring.

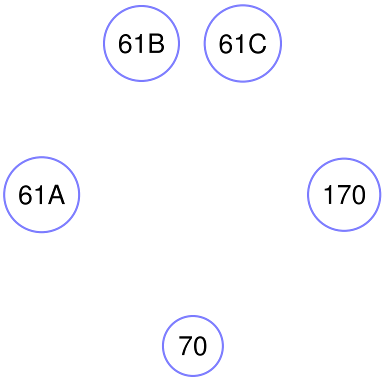




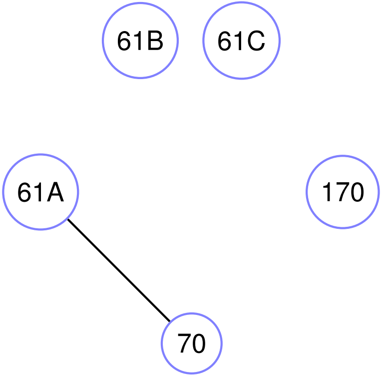
Four colors required!

Theorem: Four colors enough for maps on the plane.

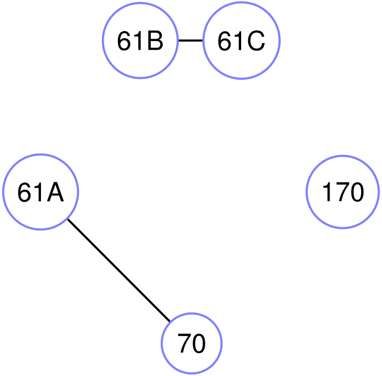
Scheduling: coloring.



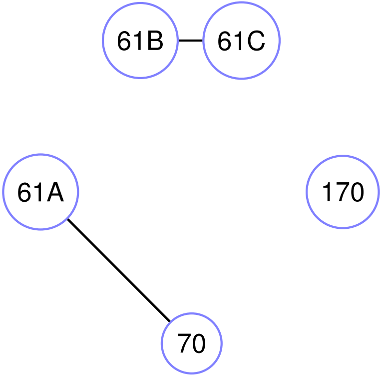
Scheduling: coloring.



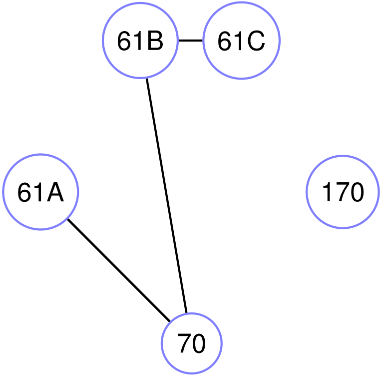
Scheduling: coloring.



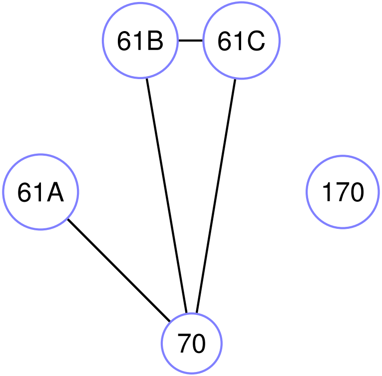
Scheduling: coloring.



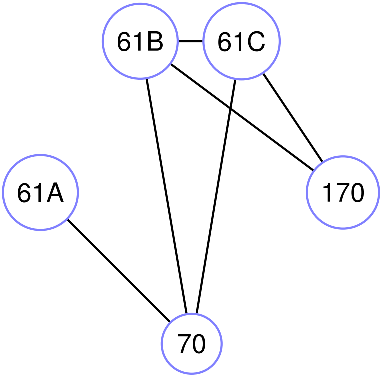
Scheduling: coloring.



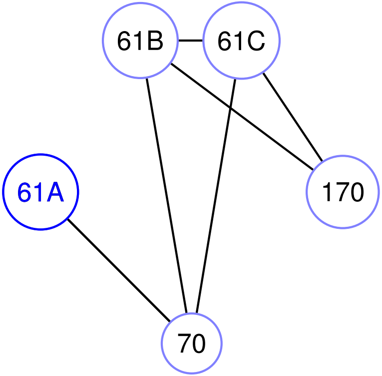
Scheduling: coloring.



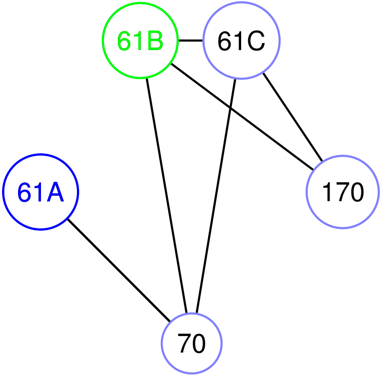
Scheduling: coloring.



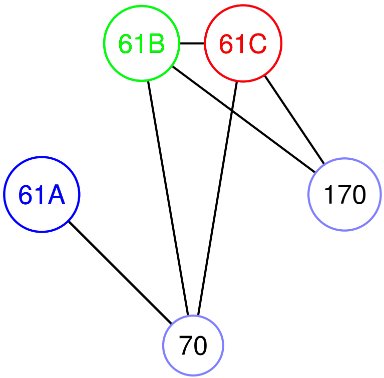
Scheduling: coloring.



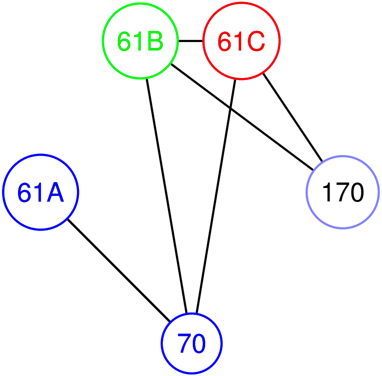
Scheduling: coloring.



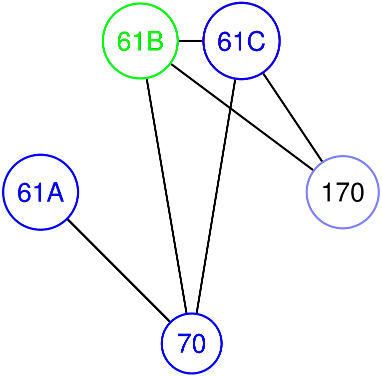
Scheduling: coloring.



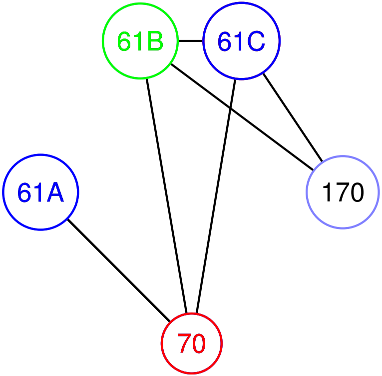
Scheduling: coloring.



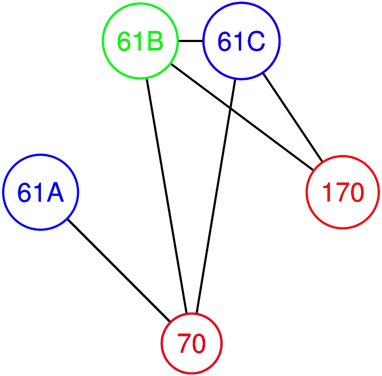
Scheduling: coloring.



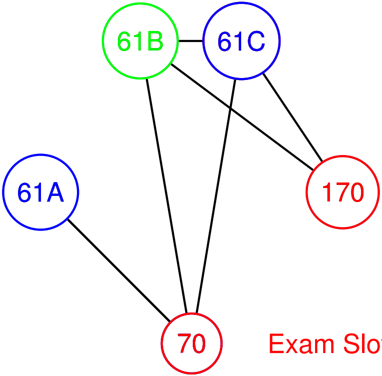
Scheduling: coloring.



Scheduling: coloring.



Scheduling: coloring.

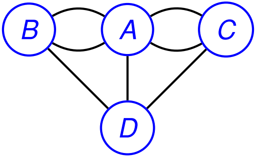


Exam Slot 1.

Exam Slot 2.

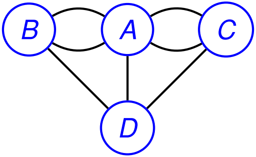
Exam Slot 3.

Graphs: formally.



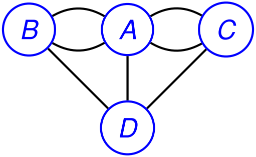
Graph:

Graphs: formally.



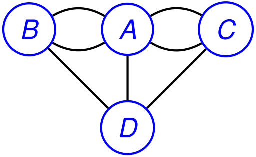
Graph: $G = (V, E)$.

Graphs: formally.



Graph: $G = (V, E)$.
  V - set of vertices.

Graphs: formally.

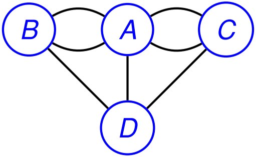


Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

Graphs: formally.



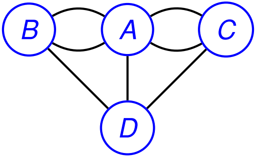
Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ -

Graphs: formally.



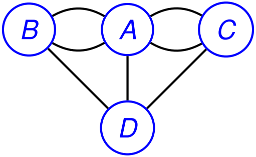
Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

Graphs: formally.



Graph: $G = (V, E)$.

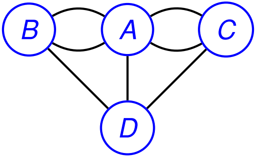
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}$

Graphs: formally.



Graph: $G = (V, E)$.

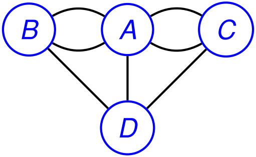
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}$

Graphs: formally.



Graph: $G = (V, E)$.

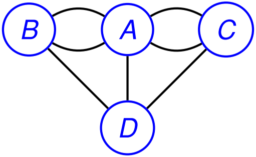
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\},$

Graphs: formally.



Graph: $G = (V, E)$.

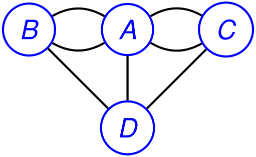
V - set of vertices.

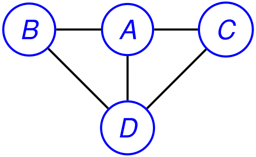
$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

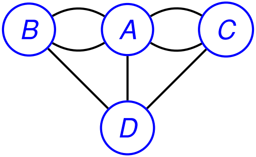
$\{A, B, C, D\}$

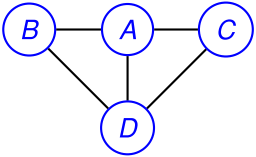
$E \subseteq V \times V$ - set of edges.

$\{\{A,B\},\{A,B\},\{A,C\},\{A,C\},\{B,D\},\{A,D\},\{C,D\}\}$.

For CS 70, usually simple graphs.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

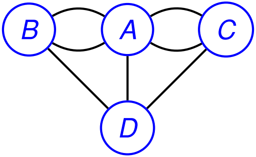
$E \subseteq V \times V$ - set of edges.

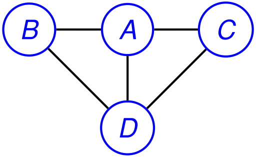
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

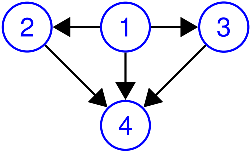
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

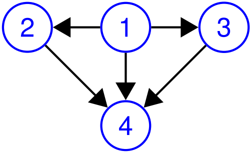
Multigraph above.

Directed Graphs



$G = (V, E)$.

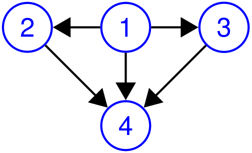
Directed Graphs



$G = (V, E)$.

V - set of vertices.

Directed Graphs

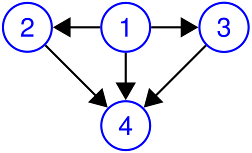


$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

Directed Graphs



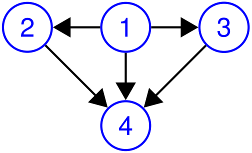
$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

Directed Graphs



$G = (V, E)$.

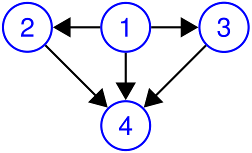
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1, 2),$

Directed Graphs



$G = (V, E)$.

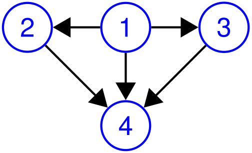
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1, 2), (1, 3),$

Directed Graphs



$G = (V, E)$.

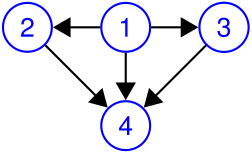
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1, 2), (1, 3), (1, 4),$

Directed Graphs



$G = (V, E)$.

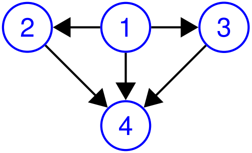
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2), (1,3), (1,4), (2,4), (3,4)\}$

Directed Graphs



$G = (V, E)$.

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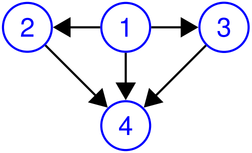
One way streets.

Directed Graphs



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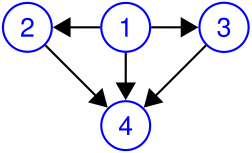
One way streets.
Tournament:

Directed Graphs



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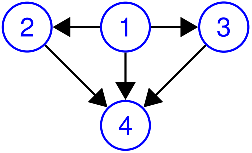
One way streets.
Tournament: 1 beats 2,

Directed Graphs



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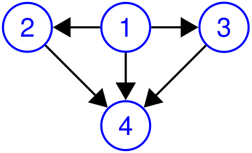
One way streets.
Tournament: 1 beats 2, ...
Precedence:

Directed Graphs



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One way streets.
Tournament: 1 beats 2, ...
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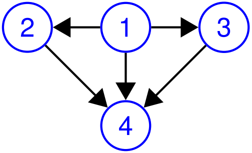
Directed Graphs



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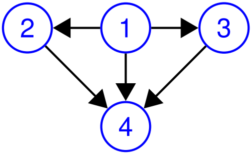

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Tournament: 1 beats 2, ...
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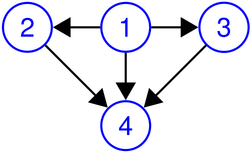
Social Network:

Directed Graphs



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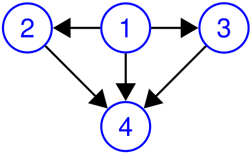
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed?

Directed Graphs



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Tournament: 1 beats 2, ...
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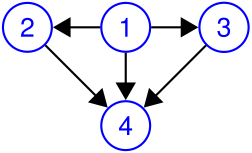
Social Network: Directed? Undirected?

Directed Graphs



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Tournament: 1 beats 2, ...
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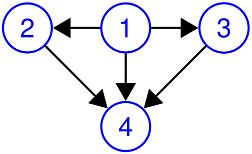
Social Network: Directed? Undirected?
Friends.

Directed Graphs



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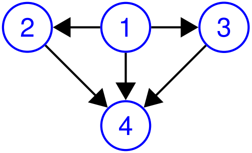
Social Network: Directed? Undirected?
Friends. Undirected.

Directed Graphs



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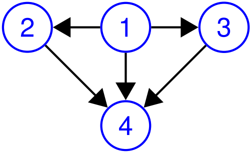
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes.

Directed Graphs



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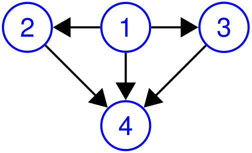
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes. Directed.

Directed Graphs



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Precedence: 1 is before 2, ..

Social Network: Directed? Undirected?
Friends. Undirected.
Likes. Directed.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

Graph Concepts and Definitions.

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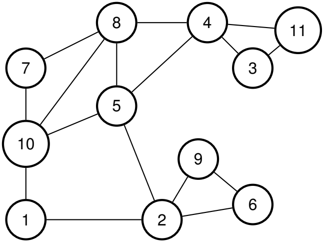
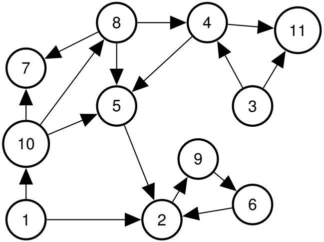
neighbors, adjacent, degree, incident, in-degree, out-degree



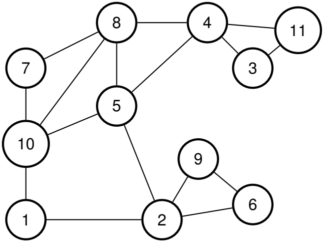


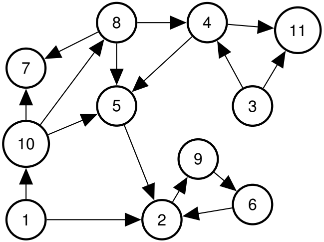
Neighbors of 10?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



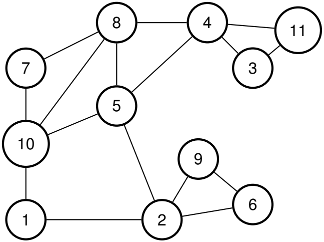


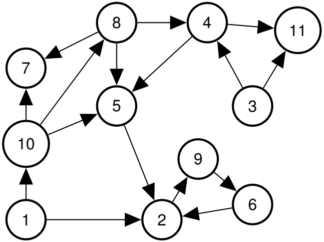
Neighbors of 10? 1,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



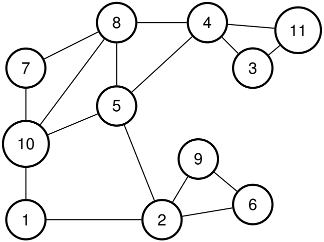


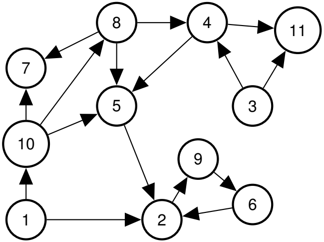
Neighbors of 10? 1,5,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



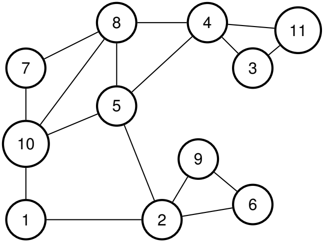


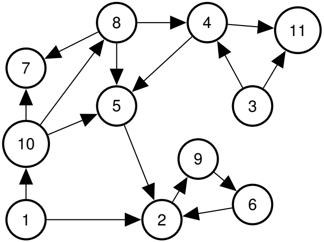
Neighbors of 10? 1,5,7,

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



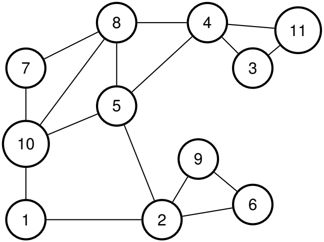


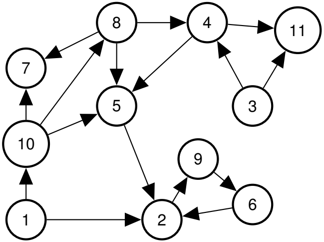
Neighbors of 10? 1,5,7, 8.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





Neighbors of 10? 1,5,7, 8.

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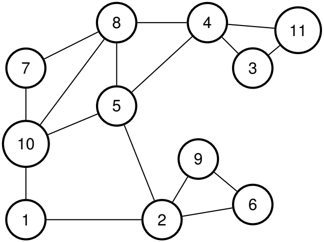

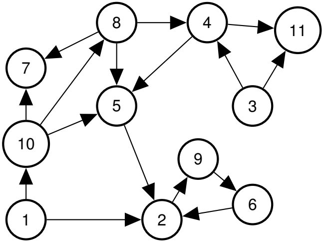

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Edge $\{10, 5\}$ is incident to

Graph Concepts and Definitions.

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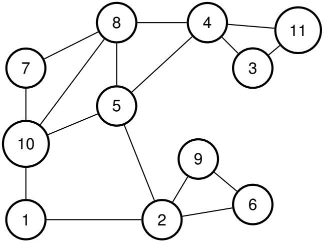

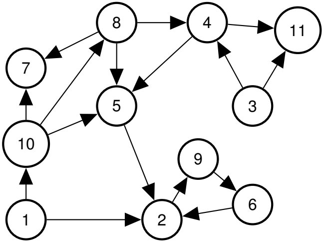

Neighbors of 10? 1,5,7, 8.

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Edge $\{10, 5\}$ is incident to vertex 10 and vertex 5.

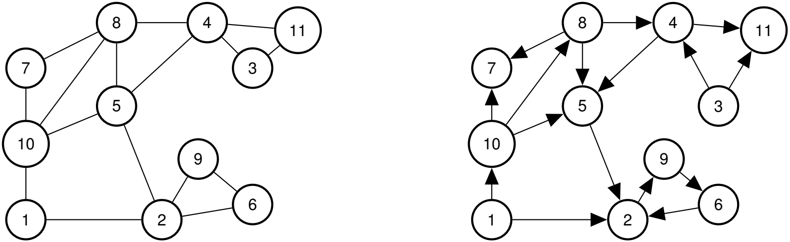
Edge $\{u, v\}$ is incident to u and v.

Degree of vertex 1?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



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Degree of vertex 1? 2

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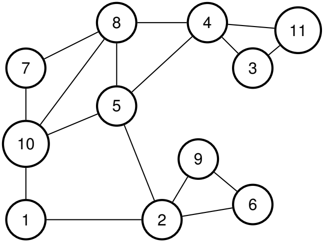

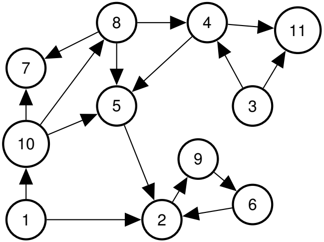

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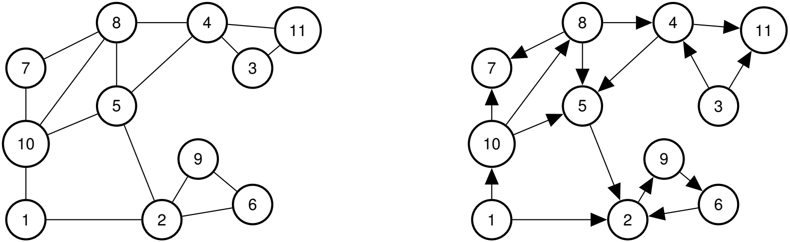
Degree of vertex 1? 2

Degree of vertex u is number of incident edges.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



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Equals number of neighbors in simple graph.

Graph Concepts and Definitions.

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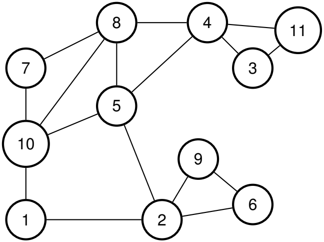

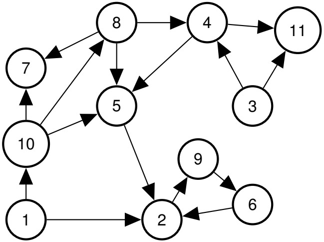

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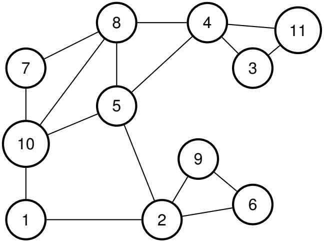

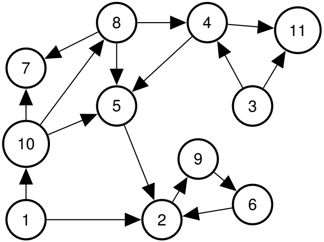

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Degree of vertex 1? 2

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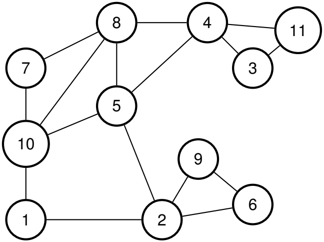
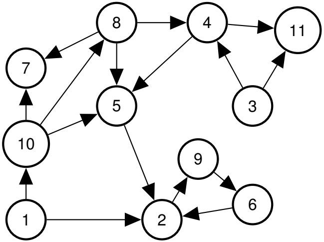
Directed graph?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree





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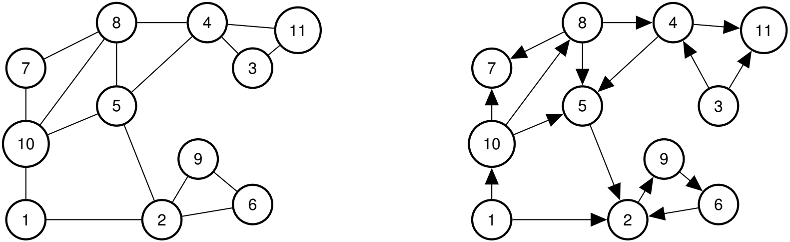
Directed graph?

In-degree of 10?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



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Directed graph?

In-degree of 10? 1

Graph Concepts and Definitions.

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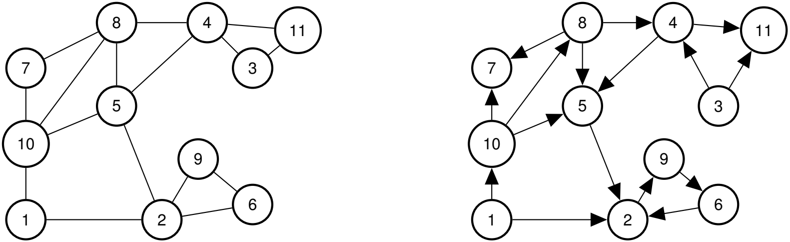
neighbors, adjacent, degree, incident, in-degree, out-degree



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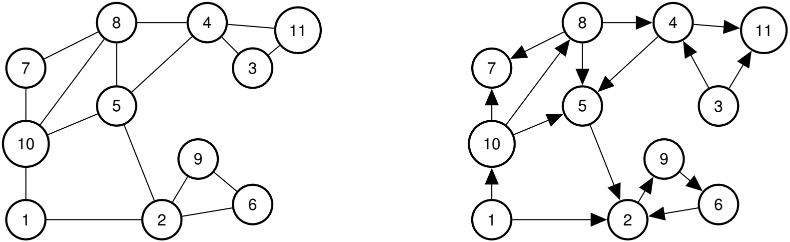
Directed graph?

In-degree of 10? 1 Out-degree of 10?

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



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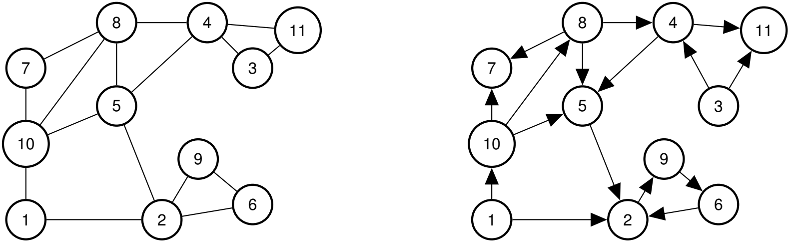
Directed graph?

In-degree of 10? 1   Out-degree of 10? 3

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



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Graph Concepts and Definitions.

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Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree

Graph Concepts and Definitions.

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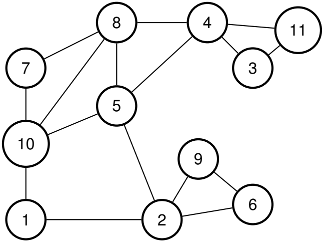

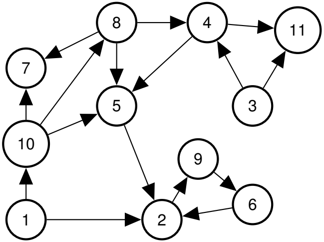

Edge $(8,5)$ **is incident to:**

(A) Vertex 8.
(B) Vertex 5.
(C) Edge $(8,5)$.
(D) Edge $(8,4)$.
(E) Vertex 10.

Graph Concepts and Definitions.

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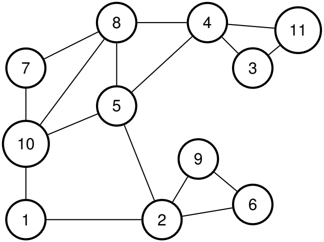

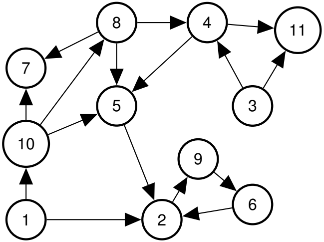

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Graph Concepts and Definitions.

Graph: $G = (V, E)$

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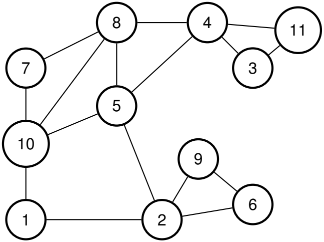

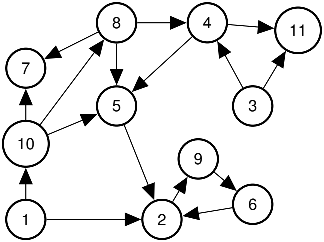

Edge $(8, 5)$ **is incident to:**

(A) Vertex 8.
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(C) Edge $(8, 5)$.
(D) Edge $(8, 4)$.
(E) Vertex 10.

(A) and (B) are true.

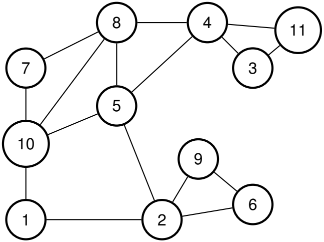
The degree of a vertex is:

(A) The number of edges incident to it.
(B) The number of neighbors of v.
(C) Is the number of vertices in its connected component.

Graph Concepts and Definitions.

Graph: $G = (V, E)$

neighbors, adjacent, degree, incident, in-degree, out-degree



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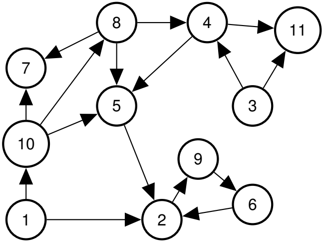

The degree of a vertex is:

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(C) Is the number of vertices in its connected component.

(A) and (B) are true.

Sum of degrees?

The sum of the vertex degrees is equal to

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(A) the total number of vertices, $|V|$.
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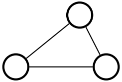
(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

(A) and (B) are false. (C) is a fine response to a poll with no correct answers.

Not (A)!



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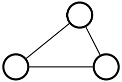
Not (A)! Triangle.



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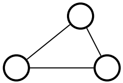

Not (A)! Triangle.
Not (B)!

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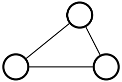

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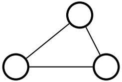

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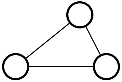

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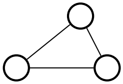

Not (A)! Triangle.
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What? For triangle number of edges is 3, the sum of degrees is 6.

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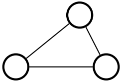

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Could sum always be...

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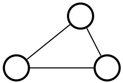

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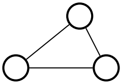

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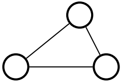

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(A) $2|E|$? ..
(B) $2|V|$?
(A) is true.

Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

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Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

Recall:

Quick Proof of an Equality.

The sum of the vertex degrees is equal to ??

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edge, (u, v), is incident to endpoints, u and v.

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degree of u number of edges incident to u

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Let's count incidences in two ways.

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How many incidences does each edge contribute?

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Total Incidences?

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Quick Proof of an Equality.

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What is degree v? Incidences corresponding to v!

Quick Proof of an Equality.

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Total Incidences? The sum over vertices of degrees!

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Thm: Sum of vertex degress is $2|E|$.

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$\sum_{v \in V} d(v) = 2|E|$.

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Let's count incidences in two ways.

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Total Incidences? The sum over vertices of degrees!

Thm: Sum of vertex degress is $2|E|$.
$\sum_{v \in V} d(v) = 2|E|$.

Poll: Proof of "handshake" lemma.

What's true?

(A) Number of edge-vertex incidences for an edge e is 2.

Poll: Proof of “handshake” lemma.

What’s true?

(A) Number of edge-vertex incidences for an edge e is 2.
(B) Total number of edge-vertex incidences is $|V|$.

Poll: Proof of “handshake” lemma.

What’s true?

(A) Number of edge-vertex incidences for an edge e is 2.
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(F) Total number of edge-vertex incidences is the sum of the degrees.

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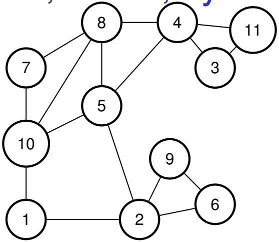
(E) Sum of degrees is $2|E|$.
(F) Total number of edge-vertex incidences is the sum of the degrees.

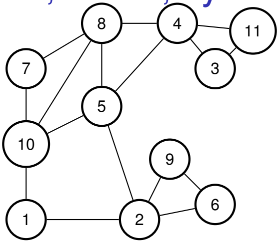
(A),(C), (D), (E), and (F).

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Paths, walks, cycles, tour.



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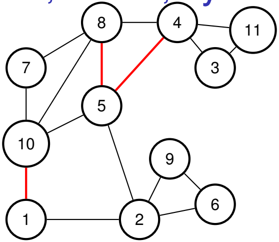
Path?

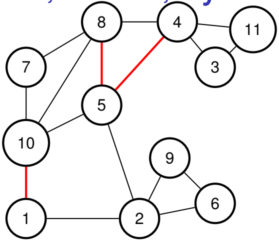
Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Path? $\{1, 10\}$, $\{8, 5\}$, $\{4, 5\}$?

Paths, walks, cycles, tour.



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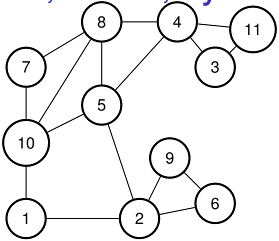

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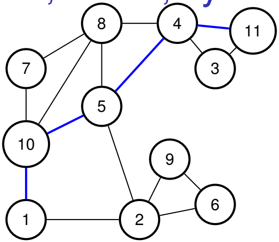
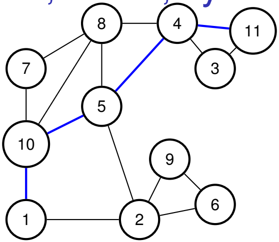


A path in a graph is a sequence of edges.

Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$? No!

Path? $\{1,10\}$, $\{10,5\}$, $\{5,4\}$, $\{4,11\}$?

Paths, walks, cycles, tour.



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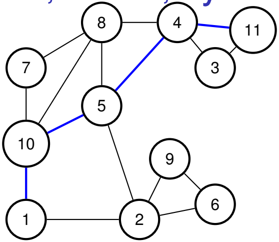


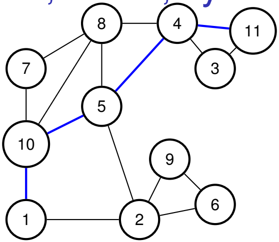
A path in a graph is a sequence of edges.

Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$? No!

Path? $\{1,10\}$, $\{10,5\}$, $\{5,4\}$, $\{4,11\}$? Yes!

Path: $(v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)$.

Paths, walks, cycles, tour.



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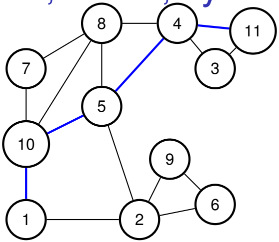
Path: $(v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)$.

Quick Check!

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$? No!

Path? $\{1,10\}$, $\{10,5\}$, $\{5,4\}$, $\{4,11\}$? Yes!

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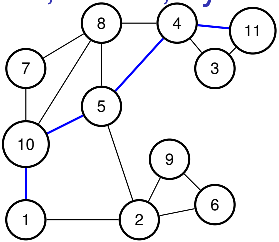
Quick Check! Length of path?

Paths, walks, cycles, tour.



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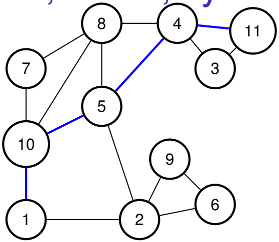

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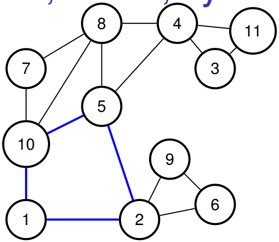

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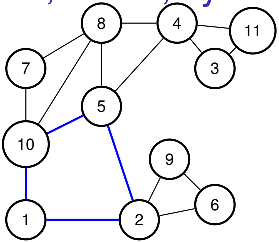
Paths, walks, cycles, tour.



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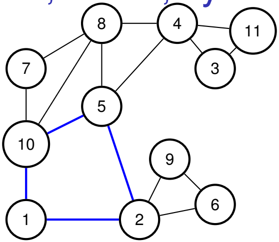

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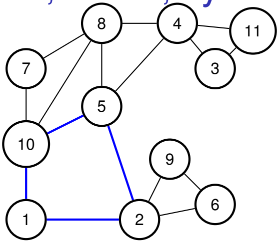

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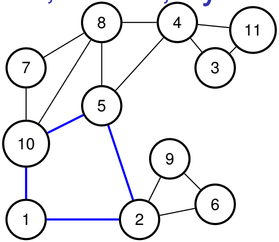
Paths, walks, cycles, tour.



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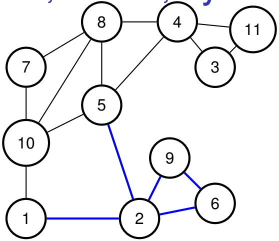

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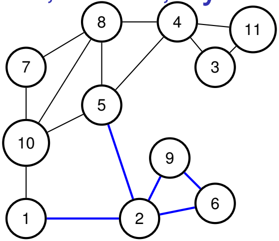

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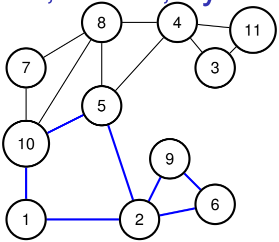

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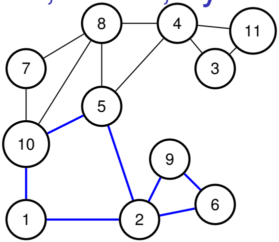

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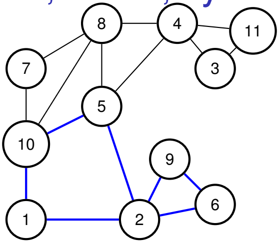

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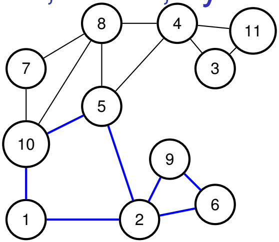

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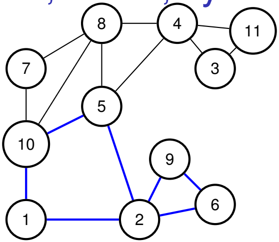
Paths, walks, cycles, tour.



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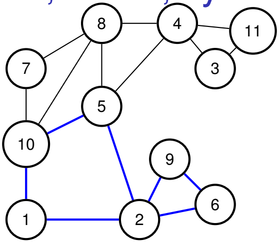

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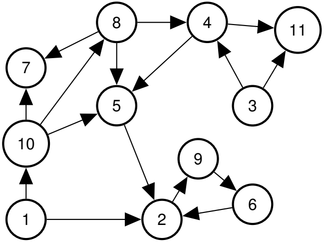

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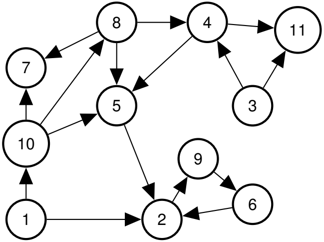

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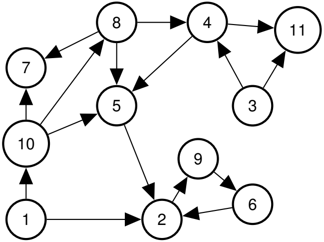

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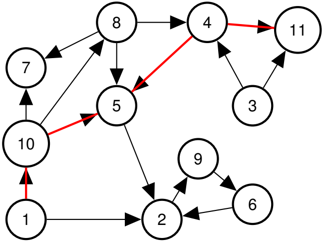

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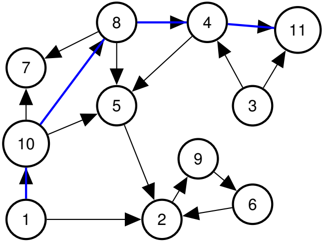

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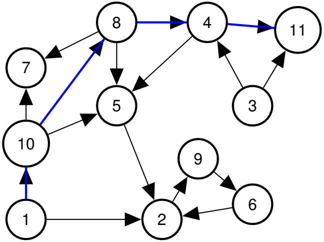

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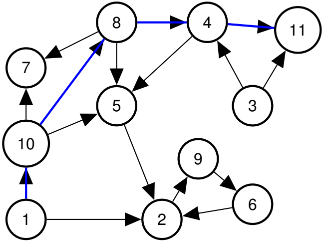

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Paths, walks,

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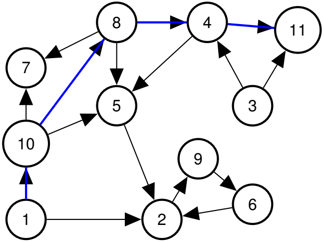

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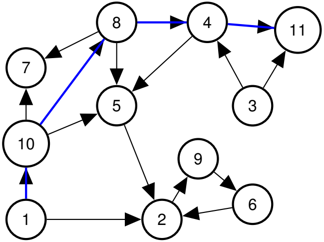

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Paths, walks, cycles, tours

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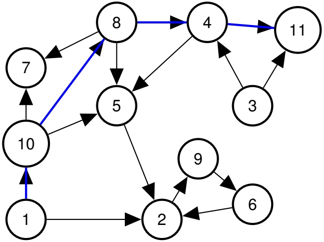

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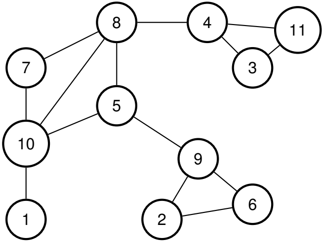
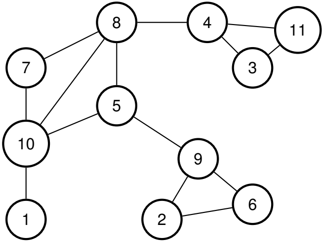
Paths, walks, cycles, tours ... are analagous to undirected now.

Connectivity: undirected graph.



u and v are connected if there is a path between u and v.

Connectivity: undirected graph.



u and *v* are connected if there is a path between *u* and *v*.

A graph is connected if all pairs of vertices are connected.

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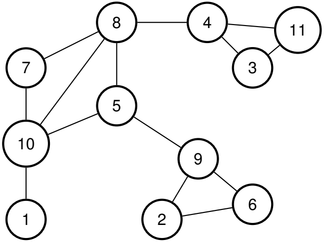

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If a vertex *x* is connected to every other vertex.

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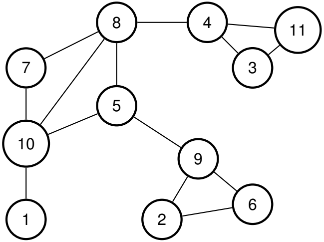

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Is graph connected?

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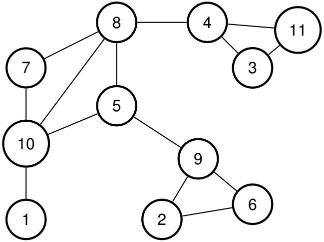

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Is graph connected? Yes?

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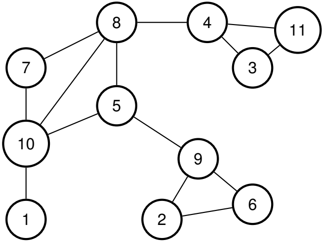

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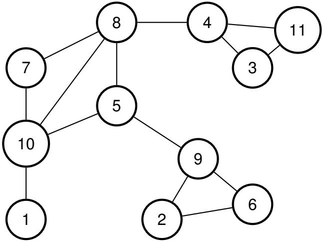

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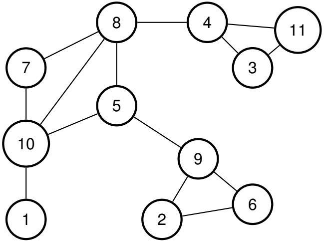
Is graph connected? Yes? No?

Proof:

Connectivity: undirected graph.



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Proof: Use path from u to x

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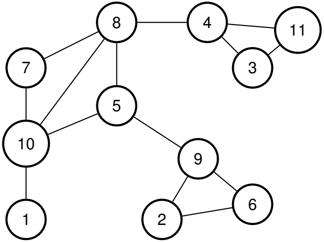

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If a vertex *x* is connected to every other vertex.
Is graph connected? Yes? No?

Proof: Use path from *u* to *x* and then from *x* to *v*.

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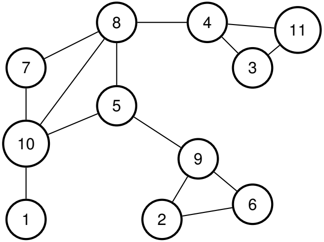

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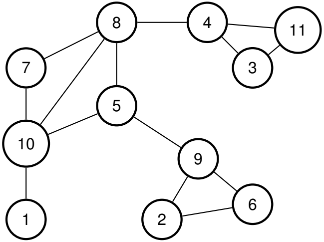
Proof: Use path from *u* to *x* and then from *x* to *v*.
Gives "walk" between *u* and *v*.

Connectivity: undirected graph.



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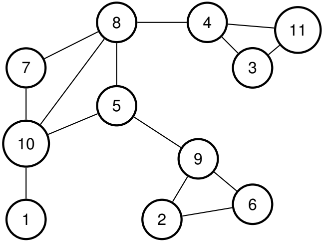

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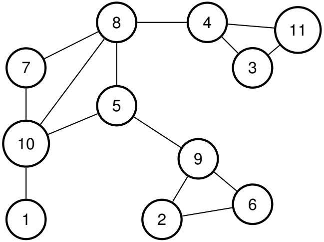
May not be simple!

Connectivity: undirected graph.



u and v are connected if there is a path between u and v.

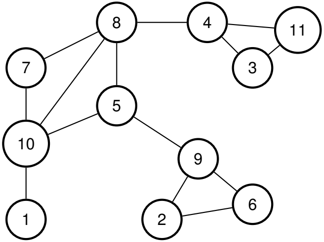
A graph is connected if all pairs of vertices are connected.

If a vertex x is connected to every other vertex.
Is graph connected? Yes? No?

Proof: Use path from u to x and then from x to v.
Gives “walk” between u and v. □

May not be simple!
Either modify definition of connected to walk.

Connectivity: undirected graph.



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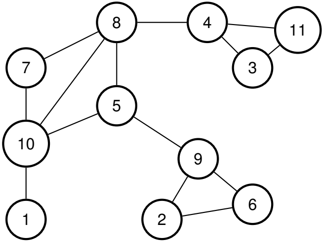

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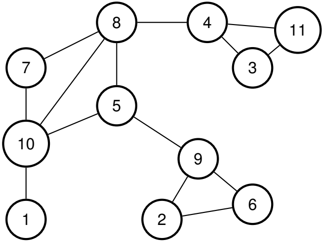

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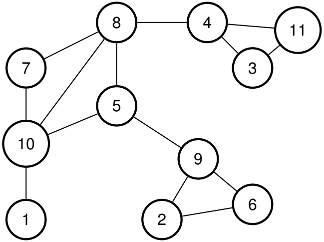
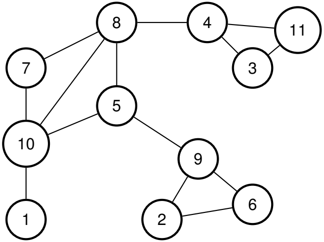
Connected Components: Quiz.



Is graph above connected?

Connected Components: Quiz.



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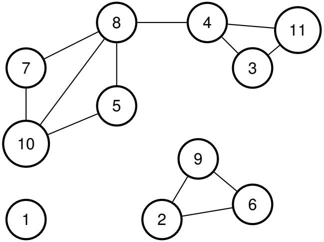

Is graph above connected? Yes!

How about now?

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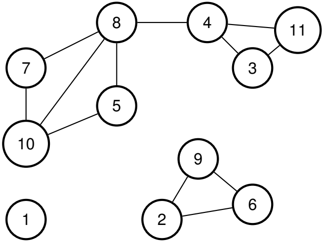

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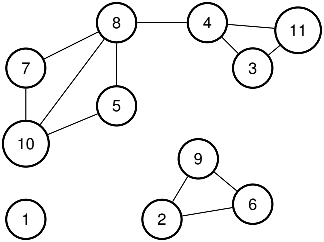

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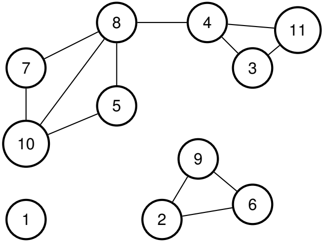

Is graph above connected? Yes!

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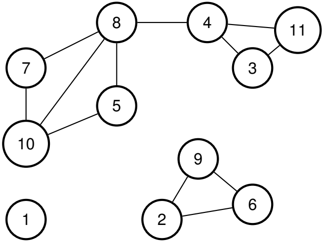
Connected Components? $\{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}$.

Connected Components: Quiz.



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Connected component - maximal set of connected vertices.

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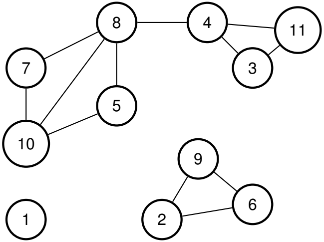

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Quick Check: Is $\{10, 7, 5\}$ a connected component?

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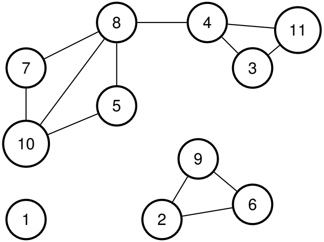

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Quick Check: Is $\{10, 7, 5\}$ a connected component? No.

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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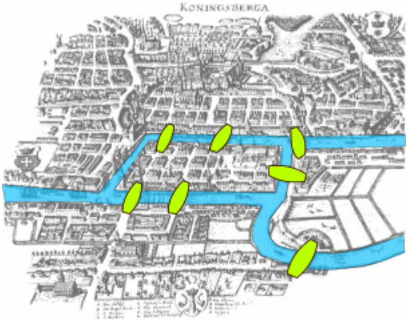

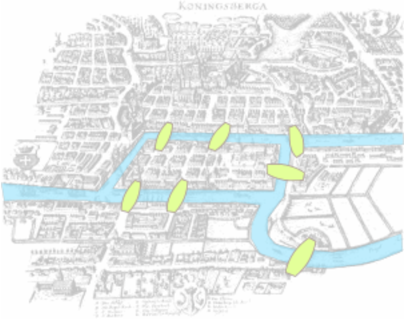

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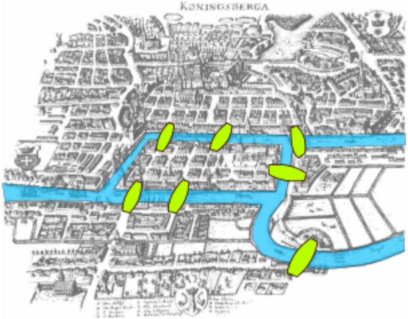

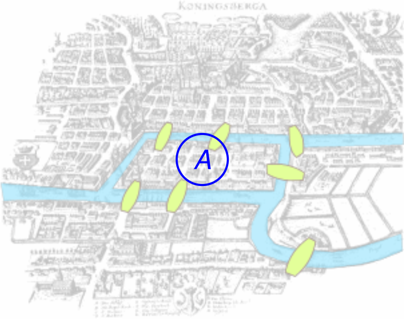

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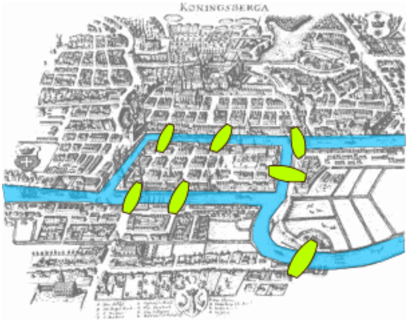

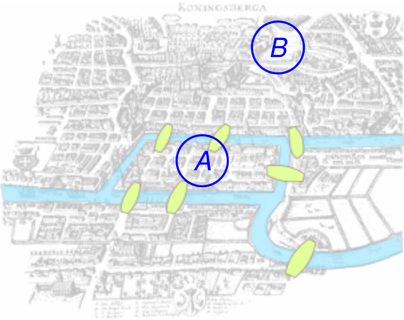

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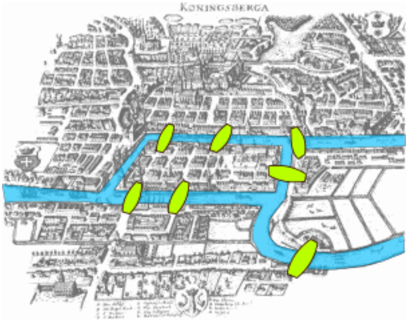

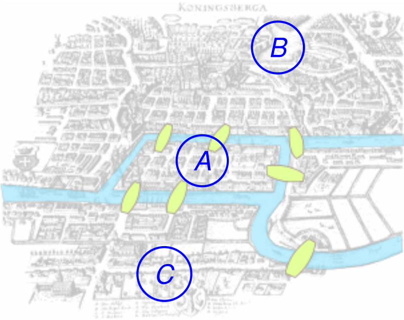

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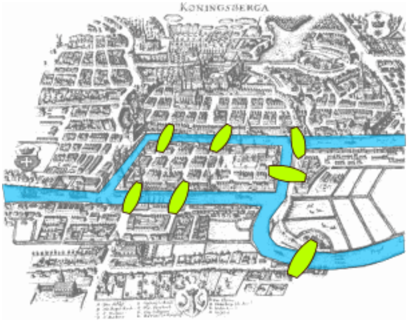

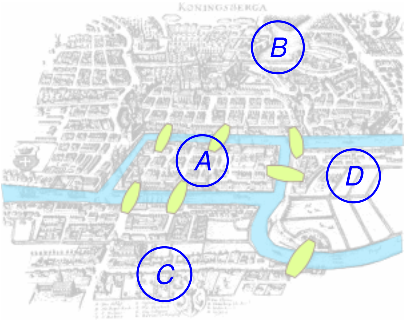

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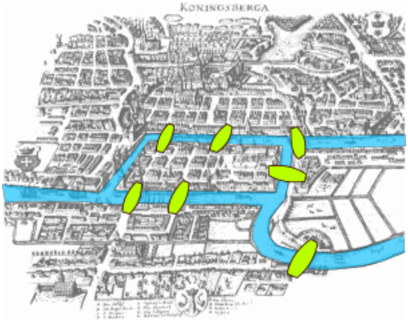

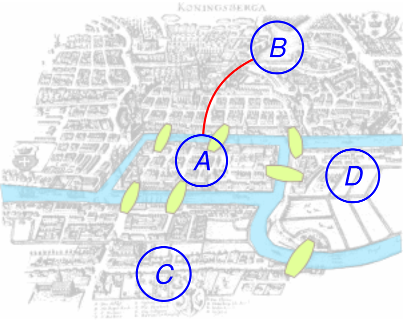

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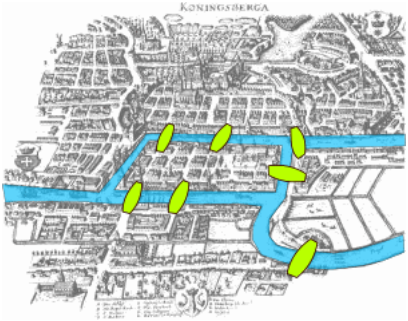

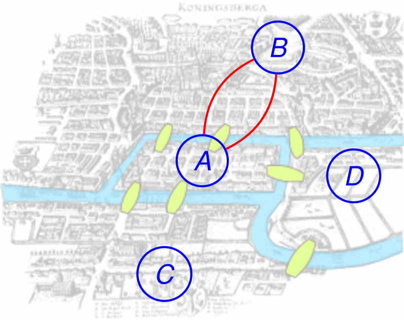

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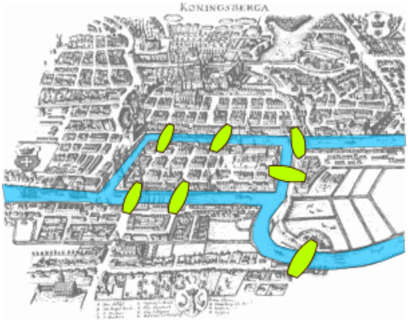

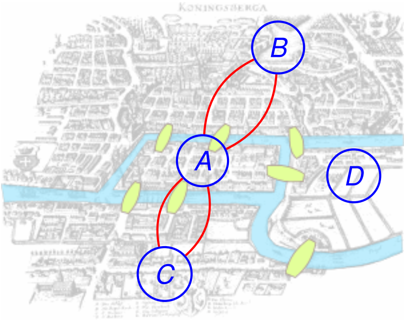

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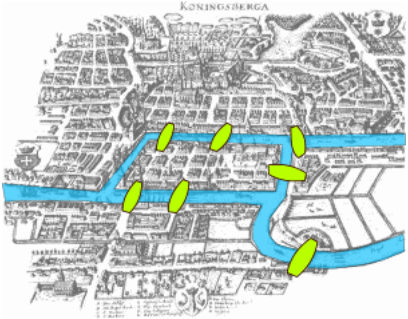

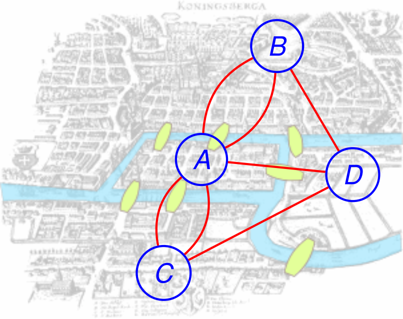

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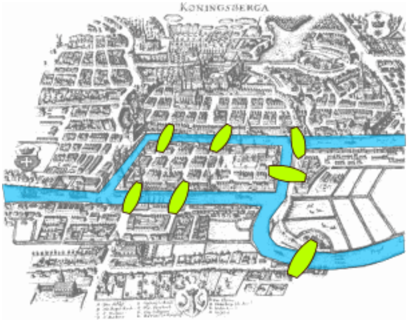

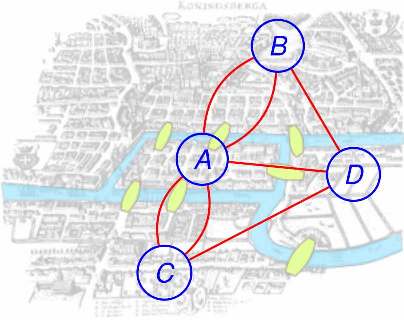

Can you draw a tour in the graph where you visit each edge once?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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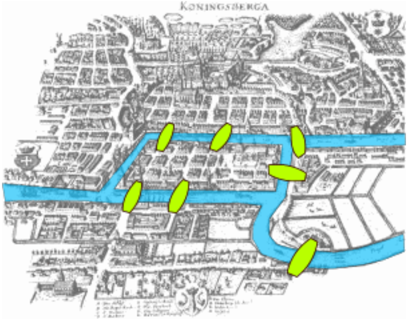

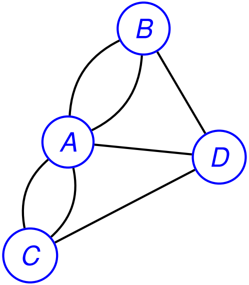

Can you draw a tour in the graph where you visit each edge once?
Yes?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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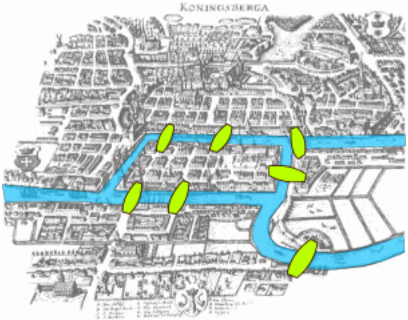

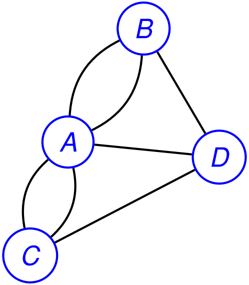

Can you draw a tour in the graph where you visit each edge once?
Yes? No?

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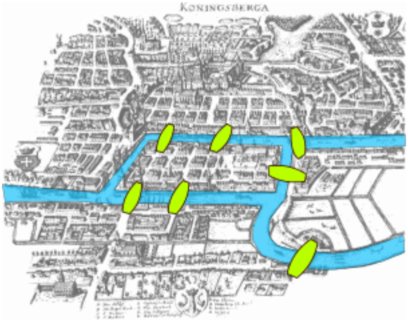

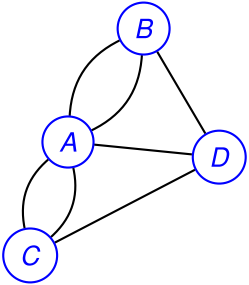

Can you draw a tour in the graph where you visit each edge once?
Yes? No?
We will see!

Eulerian Tour

Eulerian Tour visits every vertex using each edge exactly once.

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Theorem: Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

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Eulerian Tour is connected so graph is connected.

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Proof of only if: Eulerian $\Longrightarrow$ connected and all even degree.

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Tour enters and leaves vertex v on each visit.

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Tour enters and leaves vertex v on each visit.
Uses two incident edges per visit.

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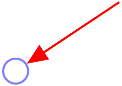

When you enter,

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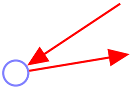

When you enter, you can leave.

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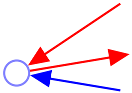

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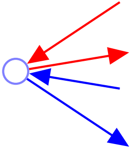

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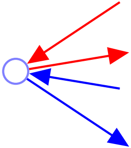

When you enter, you can leave.
For starting node,

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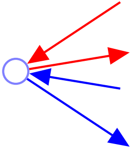

When you enter, you can leave.
For starting node, tour leaves first

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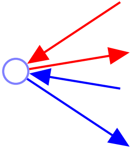

When you enter, you can leave.
For starting node, tour leaves first ....then enters at end.

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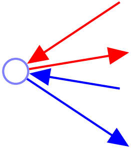
Uses two incident edges per visit. Tour uses all incident edges.
Therefore v has even degree. □



When you enter, you can leave.
For starting node, tour leaves first ....then enters at end.

Eulerian Tour

Eulerian Tour visits every vertex using each edge exactly once.

Theorem: Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.
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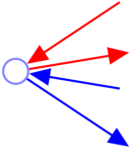

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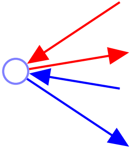

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(Timestamp: 4:02).

Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm.

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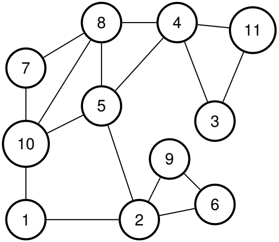

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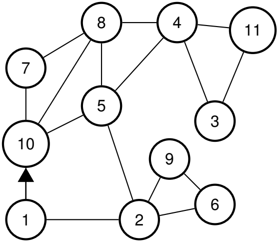

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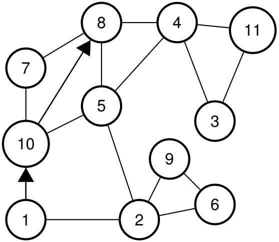

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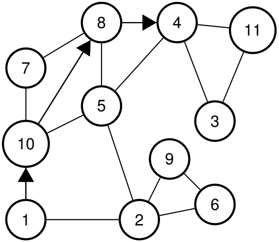

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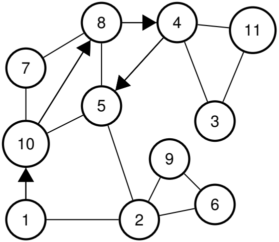

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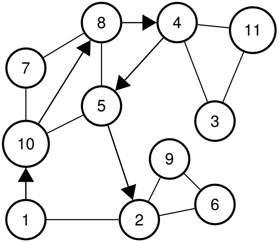

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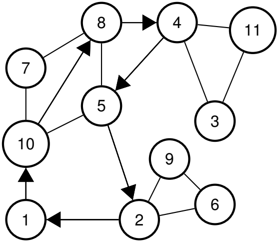

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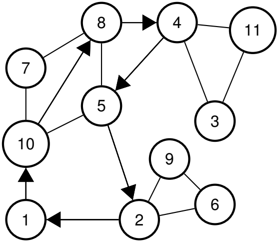

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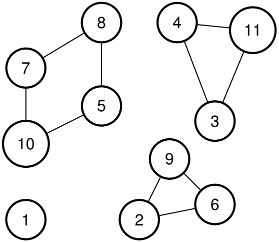

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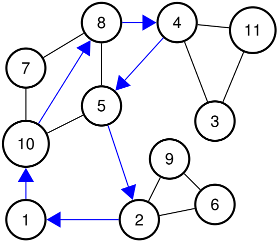

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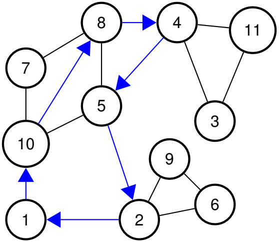

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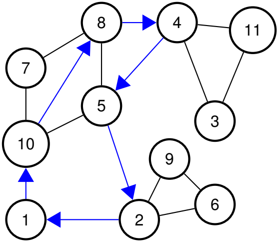

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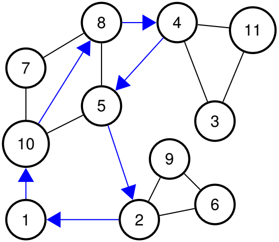

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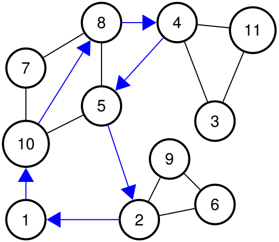

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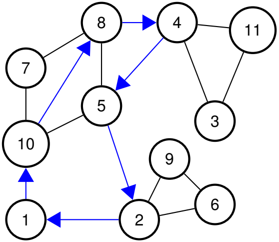

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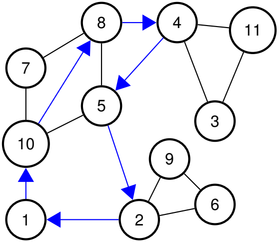

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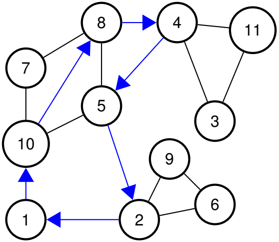

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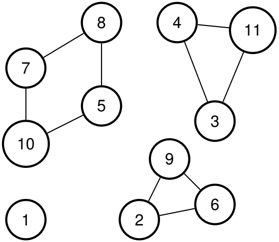

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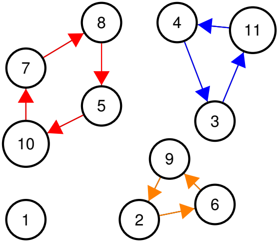

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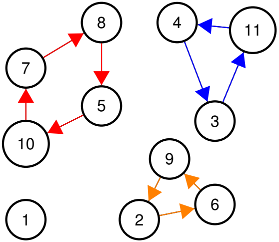

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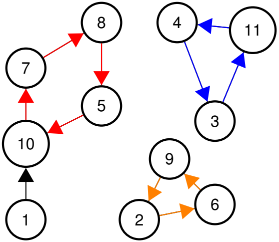

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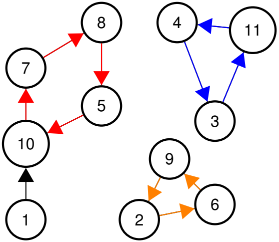

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   1,10,7,8,5,10

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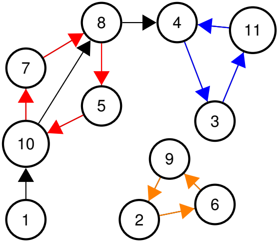

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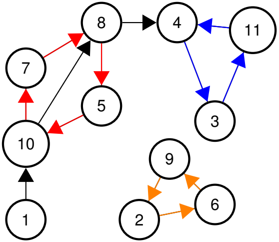

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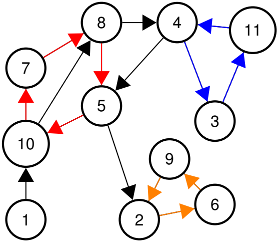

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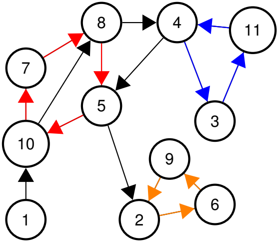

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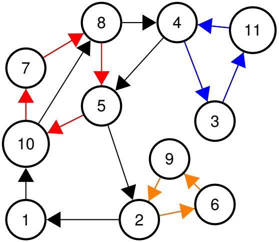

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Recursive/Inductive Algorithm.

1. Take a walk from arbitrary node v, until you get back to v.

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Claim: Do get back to v!

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Proof of Claim: Even degree.

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Resulting graph may be disconnected. (Removed edges!)

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Why is there a v_i in C?

Recursive/Inductive Algorithm.

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Claim: Do get back to v!
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Why is there a v_i in C?
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1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

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(C) There is no hotel california in this graph.
(D) After removing a set of edges E' in a connected graph, every connected component is incident to an edge in E'
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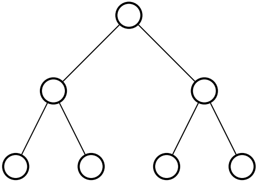
Only (F) is false.

A Tree, a tree.

Graph $G = (V, E)$.
Binary Tree!



More generally.

Trees.

Definitions:

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A connected graph without a cycle.

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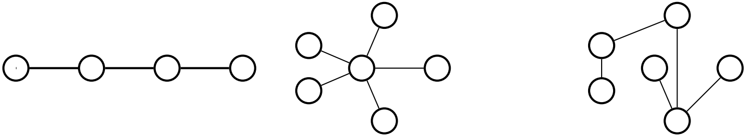

no cycle and connected?

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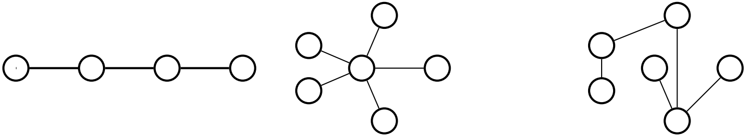

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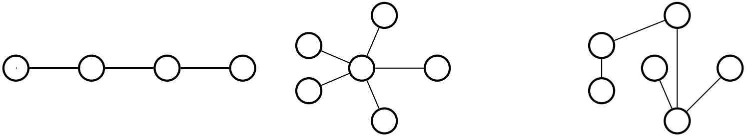

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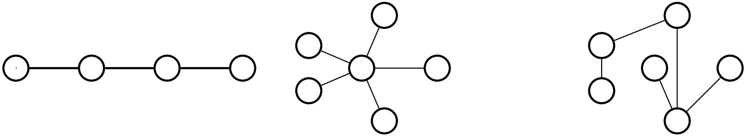

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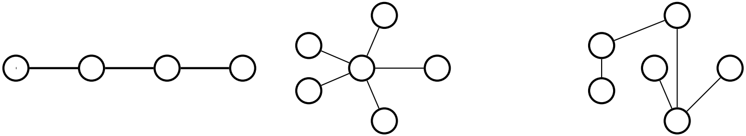

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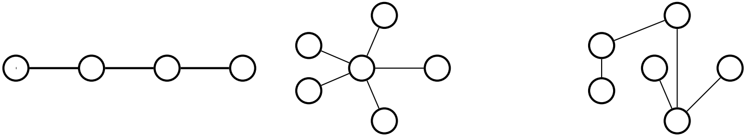

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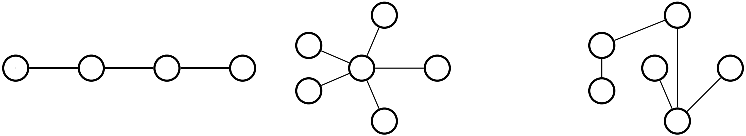

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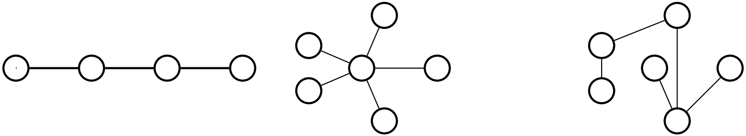

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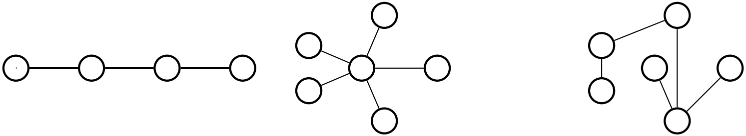

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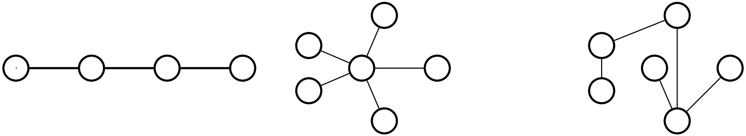

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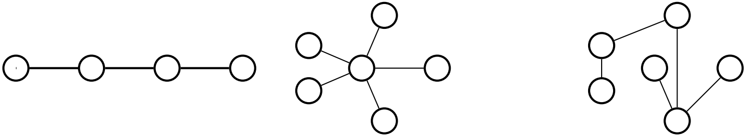

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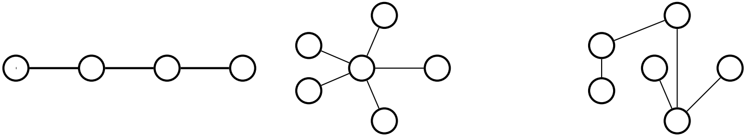

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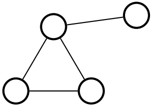

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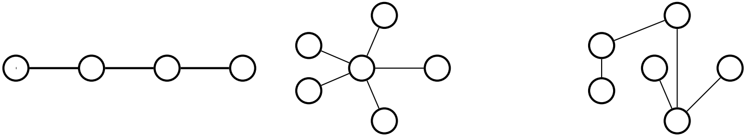

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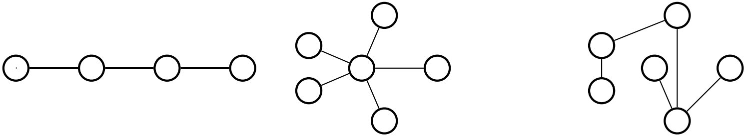

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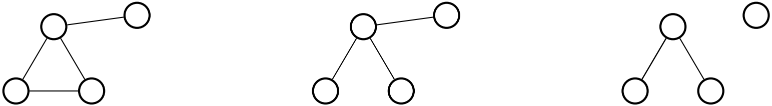

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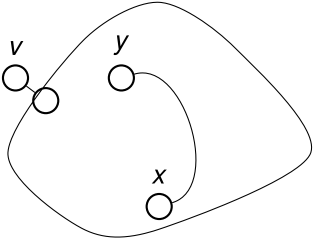

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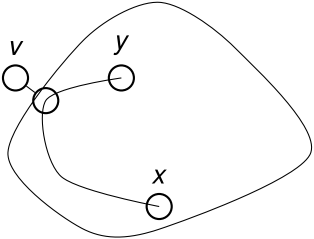

Proof of only if.

Thm:
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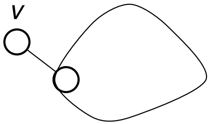
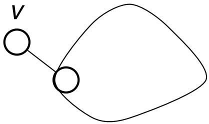
"G is connected and has no cycles."



Proof of $\implies$:

Proof of only if.



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Proof of $\implies$ **:** By induction on $|V|$.

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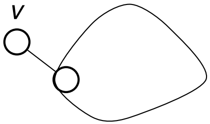

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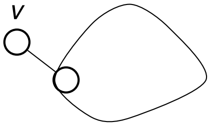

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Proof of $\implies$: By induction on $|V|$.
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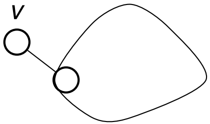

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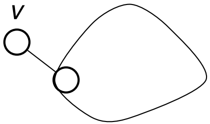

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Induction Step:
Claim: There is a degree 1 node.

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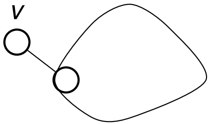

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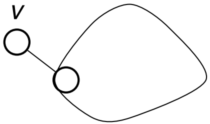
Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected $\implies$ every vertex degree ≥ 1.

Proof of only if.



Thm:
"G connected and has $|V|-1$ edges" $\Longrightarrow$
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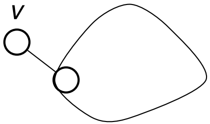
Proof of $\Longrightarrow$: By induction on $|V|$.
Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected $\Longrightarrow$ every vertex degree ≥ 1.
Sum of degrees is $2|E| = 2(|V|-1) = 2|V|-2$

Proof of only if.



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Induction Step:
Claim: There is a degree 1 node.
Proof: First, connected $\implies$ every vertex degree ≥ 1.
Sum of degrees is $2|E|=2(|V|-1)=2|V|-2$
Average degree $(2|V|-2)/|V|=2-(2/|V|)$.

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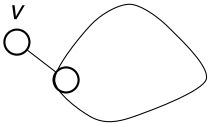

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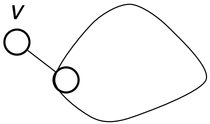

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Base Case: $|V| = 1$. $0 = |V| - 1$ edges and has no cycles.

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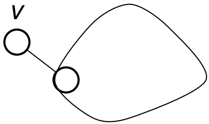
Claim: There is a degree 1 node.
Proof: First, connected $\implies$ every vertex degree ≥ 1.
Sum of degrees is $2|E| = 2(|V| - 1) = 2|V| - 2$
Average degree $(2|V| - 2)/|V| = 2 - (2/|V|)$. Must be a degree 1 vertex.
Cuz not everyone is bigger than average!

Proof of only if.



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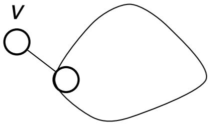

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By degree 1 removal lemma, $G-v$ is connected.

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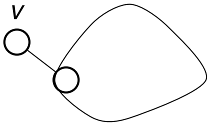

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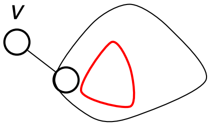

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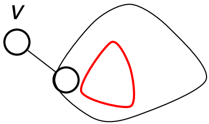

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And no cycle in G since degree 1 cannot participate in cycle.

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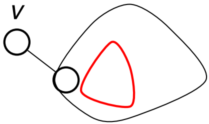

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Thm:
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Proof:
Walk from a vertex using untraversed edges.

Proof of if

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Until get stuck.

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Claim: Degree 1 vertex.

Proof of if

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Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.
Proof of Claim:
Can't visit more than once since no cycle.

Proof of if

Thm:
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Proof:
Walk from a vertex using untraversed edges.
Until get stuck.
Claim: Degree 1 vertex.
Proof of Claim:
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Entered.

Proof of if

Thm:
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Walk from a vertex using untraversed edges.
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Can't visit more than once since no cycle.
Entered. Didn't leave.

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Removing node doesn't create cycle.

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New graph is connected.

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Removing node doesn't create cycle.
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Removing degree 1 node doesn't disconnect from Degree 1 lemma.

Proof of if

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By induction $G - v$ has $|V| - 2$ edges.
G has one more or $|V| - 1$ edges.

Proof of if

Thm:
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By induction $G - v$ has $|V| - 2$ edges.
G has one more or $|V| - 1$ edges. □

Poll: Oh tree, beautiful tree.

Let G be a connected graph with $|V| - 1$ edges.

Poll: Oh tree, beautiful tree.

Let G be a connected graph with $|V|-1$ edges.

(A) Removing a degree 1 vertex can disconnect the graph.
(B) One can use induction on smaller objects.
(C) The average degree is $2-2/|V|$.
(D) There is a hotel california: a degree 1 vertex.
(E) Everyone can be bigger than average.

Poll: Oh tree, beautiful tree.

Let G be a connected graph with $|V|-1$ edges.

(A) Removing a degree 1 vertex can disconnect the graph.
(B) One can use induction on smaller objects.
(C) The average degree is $2-2/|V|$.
(D) There is a hotel california: a degree 1 vertex.
(E) Everyone can be bigger than average.

(B), (C), (D) are true

Lecture Summary.

Graphs.

Lecture Summary.

Graphs.
  Basics.

Lecture Summary.

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Degree, Incidence, Sum of degrees is $2|E|$. Connectivity.

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Connected Component.

Lecture Summary.

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Trees: degree 1 lemma $\Longrightarrow$ equivalence of several definitions.

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Trees: degree 1 lemma $\implies$ equivalence of several definitions.
G: n vertices and $n-1$ edges and connected.
remove degree 1 vertex.
$n-1$ vertices, $n-2$ edges and connected $\implies$ acyclic.
(Ind. Hyp.)
degree 1 vertex is not in a cycle.
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