Do you remember the first lecture?

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Veritassium on Khan

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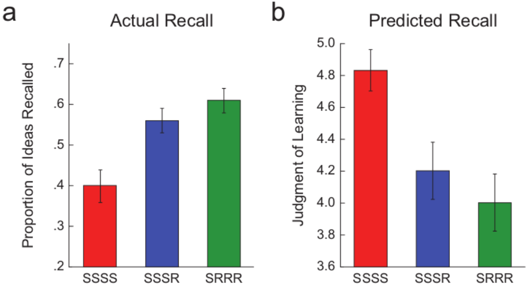

Fig. 1. Final recall (a) after repeatedly studying a text in four study periods (SSSS condition), reading a text in three study periods and then recalling it in one retrieval period (SSSR condition), or reading a text in one study period and then repeatedly recalling it in three retrieval periods (SRRR condition). Judgments of learning (b) were made on a 7-point scale, where 7 indicated that students believed they would remember material very well. The data presented in these graphs are adapted from Experiment 2 of Roediger and Karpicke (2006b). The pattern of students' metacognitive judgments of learning (predicted recall) was exactly the opposite of the pattern of students' actual long-term retention.

CS70: Lecture 9. Outline.

1. Public Key Cryptography
2. RSA system
    2.1 Efficiency: Repeated Squaring.
    2.2 Correctness: Fermat's Theorem.
    2.3 Construction.
3. Warnings.

Simple Chinese Remainder Theorem.

Simple Chinese Remainder Theorem.

My love is won.

Simple Chinese Remainder Theorem.

My love is won. Zero and One.

Simple Chinese Remainder Theorem.

My love is won. Zero and One. Nothing and nothing done.

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Find $x = a \pmod{m}$ and $x = b \pmod{n}$

Simple Chinese Remainder Theorem.

My love is won. Zero and One. Nothing and nothing done.

Find $x = a \pmod{m}$ and $x = b \pmod{n}$ where gcd(m,n)=1.

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CRT Thm: There is a unique solution $x \pmod{mn}$.

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Proof (solution exists):

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CRT Thm: There is a unique solution $x \pmod{mn}$.
Proof (solution exists):
Consider $u = n(n^{-1} \pmod{m})$.

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Consider $v = m(m^{-1} \pmod{n})$.
$v = 1 \pmod{n}$ $\quad$ $v = 0 \pmod{m}$
Let $x = au + bv$.

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Let $x = au + bv$.
$x = a \pmod{m}$

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$v = 1 \pmod{n}$ $\quad$ $v = 0 \pmod{m}$
Let $x = au + bv$.
$x = a \pmod{m}$ since $bv = 0 \pmod{m}$ and $au = a \pmod{m}$

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$x = a \pmod{m}$ since $bv = 0 \pmod{m}$ and $au = a \pmod{m}$
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Consider $v = m(m^{-1} \pmod{n})$.
$v = 1 \pmod{n}$ $\quad$ $v = 0 \pmod{m}$
Let $x = au + bv$.
$x = a \pmod{m}$ since $bv = 0 \pmod{m}$ and $au = a \pmod{m}$
$x = b \pmod{n}$ since $au = 0 \pmod{n}$ and $bv = b \pmod{n}$
This shows there is a solution. $\square$

Simple Chinese Remainder Theorem.

CRT Thm: There is a unique solution $x \pmod{mn}$.

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Proof (uniqueness):

Simple Chinese Remainder Theorem.

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Proof (uniqueness):
If not, two solutions, x and y.

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Simple Chinese Remainder Theorem.

CRT Thm: There is a unique solution $x \pmod{mn}$.

Proof (uniqueness):
If not, two solutions, x and y.

$(x - y) \equiv 0 \pmod{m}$ and $(x - y) \equiv 0 \pmod{n}$.

Simple Chinese Remainder Theorem.

CRT Thm: There is a unique solution $x \pmod{mn}$.

Proof (uniqueness):
If not, two solutions, x and y.

$(x-y) \equiv 0 \pmod{m}$ and $(x-y) \equiv 0 \pmod{n}$.
$\implies (x-y)$ is multiple of m and n

Simple Chinese Remainder Theorem.

CRT Thm: There is a unique solution $x \pmod{mn}$.

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If not, two solutions, x and y.

$(x-y) \equiv 0 \pmod{m}$ and $(x-y) \equiv 0 \pmod{n}$.
$\implies (x-y)$ is multiple of m and n
$\gcd(m,n) = 1 \implies$ no common primes in factorization m and n

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$\gcd(m,n) = 1 \implies$ no common primes in factorization m and n
$\implies mn | (x-y)$

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$\gcd(m,n) = 1 \implies$ no common primes in factorization m and n
$\implies mn|(x-y)$
$\implies x-y \geq mn$

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$\gcd(m,n) = 1 \implies$ no common primes in factorization m and n
$\implies mn|(x-y)$
$\implies x-y \geq mn \implies x,y \notin \{1,\dots,mn-1\}$.
(e.g., $m=2, n=5$, $x,y \in \{1,\dots,9\} \implies x-y < 10$.)

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Thus, only one solution modulo mn.

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If not, two solutions, x and y.

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$\implies (x - y)$ is multiple of m and n
$\gcd(m, n) = 1 \implies$ no common primes in factorization m and n
$\implies mn | (x - y)$
$\implies x - y \geq mn \implies x, y \notin \{1, \ldots, mn - 1\}$.
(e.g., $m = 2, n = 5$, $x, y \in \{1, \ldots, 9\} \implies x - y < 10$.)

Thus, only one solution modulo mn. $\square$

Isomorphisms.

Bijection:

Isomorphisms.

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$f(x) = ax \pmod{m}$ if $gcd(a, m) = 1$.

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Simplified Chinese Remainder Theorem:

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Simplified Chinese Remainder Theorem:

If $gcd(n,m) = 1$, there is unique $x \pmod{mn}$ where

$x = a \pmod{m}$ and $x = b \pmod{n}$.

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Simplified Chinese Remainder Theorem:
If $gcd(n,m) = 1$, there is unique $x \pmod{mn}$ where
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Bijection between $(a \pmod{n}, b \pmod{m})$ and $x \pmod{mn}$.

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Bijection between $(a \pmod{n}, b \pmod{m})$ and $x \pmod{mn}$.

Consider $m = 5$, $n = 9$, then if $(a,b) = (3,7)$ then $x = 43 \pmod{45}$.

Isomorphisms.

Bijection:

$f(x) = ax \pmod{m}$ if $gcd(a,m) = 1$.

Simplified Chinese Remainder Theorem:
If $gcd(n,m) = 1$, there is unique $x \pmod{mn}$ where
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Bijection between $(a \pmod{n}, b \pmod{m})$ and $x \pmod{mn}$.

Consider $m = 5$, $n = 9$, then if $(a,b) = (3,7)$ then $x = 43 \pmod{45}$.

Consider $(a', b') = (2,4)$, then $x = 22 \pmod{45}$.

Isomorphisms.

Bijection:

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If $gcd(n,m) = 1$, there is unique $x \pmod{mn}$ where
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Bijection between $(a \pmod{n}, b \pmod{m})$ and $x \pmod{mn}$.

Consider $m = 5$, $n = 9$, then if $(a,b) = (3,7)$ then $x = 43 \pmod{45}$.

Consider $(a', b') = (2,4)$, then $x = 22 \pmod{45}$.

Now consider:

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Bijection between $(a \pmod{n}, b \pmod{m})$ and $x \pmod{mn}$.

Consider $m = 5$, $n = 9$, then if $(a,b) = (3,7)$ then $x = 43 \pmod{45}$.

Consider $(a',b') = (2,4)$, then $x = 22 \pmod{45}$.

Now consider: $(a,b) + (a',b') = (0,2)$.

Isomorphisms.

Bijection:

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If $gcd(n,m) = 1$, there is unique $x \pmod{mn}$ where
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Bijection between $(a \pmod{n}, b \pmod{m})$ and $x \pmod{mn}$.

Consider $m = 5$, $n = 9$, then if $(a,b) = (3,7)$ then $x = 43 \pmod{45}$.

Consider $(a', b') = (2,4)$, then $x = 22 \pmod{45}$.

Now consider: $(a,b) + (a',b') = (0,2)$.

What is x where $x = 0 \pmod{5}$ and $x = 2 \pmod{9}$?

Isomorphisms.

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If $gcd(n,m) = 1$, there is unique $x \pmod{mn}$ where
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What is x where $x = 0 \pmod{5}$ and $x = 2 \pmod{9}$?

Try $43 + 22 = 65$

Isomorphisms.

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Consider $(a',b') = (2,4)$, then $x = 22 \pmod{45}$.

Now consider: $(a,b) + (a',b') = (0,2)$.

What is x where $x = 0 \pmod{5}$ and $x = 2 \pmod{9}$?

Try $43 + 22 = 65 = 20 \pmod{45}$.

Isomorphisms.

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Is it $0 \pmod{5}$?

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Is it $0 \pmod{5}$? Yes!

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Isomorphism:

Isomorphisms.

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Try $43 + 22 = 65 = 20 \pmod{45}$.

Is it $0 \pmod{5}$? Yes! Is it $2 \pmod{9}$? Yes!

Isomorphism:
the actions under $\pmod{5}$, $\pmod{9}$

Isomorphisms.

Bijection:

$f(x) = ax \pmod{m}$ if $gcd(a, m) = 1$.

Simplified Chinese Remainder Theorem:
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What is x where $x = 0 \pmod{5}$ and $x = 2 \pmod{9}$?

Try $43 + 22 = 65 = 20 \pmod{45}$.

Is it $0 \pmod{5}$? Yes! Is it $2 \pmod{9}$? Yes!

Isomorphism:
the actions under $\pmod{5}$, $\pmod{9}$
correspond to actions in $\pmod{45}$!

Poll

$x = 5 \mod 7$ **and** $x = 5 \mod 6$
$y = 4 \mod 7$ **and** $y = 3 \mod 6$

Poll

$x = 5 \mod 7$ **and** $x = 5 \mod 6$
$y = 4 \mod 7$ **and** $y = 3 \mod 6$

What's true?

Poll

$x = 5 \mod 7$ **and** $x = 5 \mod 6$
$y = 4 \mod 7$ **and** $y = 3 \mod 6$

What's true?

(A) $x + y = 2 \mod 7$

Poll

$x = 5 \mod 7$ **and** $x = 5 \mod 6$
$y = 4 \mod 7$ **and** $y = 3 \mod 6$

What's true?

(A) $x + y = 2 \mod 7$
(B) $x + y = 2 \mod 6$

Poll

$x = 5 \mod 7$ **and** $x = 5 \mod 6$
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What's true?

(A) $x + y = 2 \mod 7$
(B) $x + y = 2 \mod 6$
(C) $xy = 3 \mod 6$

Poll

$x = 5 \mod 7$ **and** $x = 5 \mod 6$
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What's true?

(A) $x + y = 2 \mod 7$
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(C) $xy = 3 \mod 6$
(D) $xy = 6 \mod 7$

Poll

$x = 5 \mod 7$ **and** $x = 5 \mod 6$
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What's true?

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(C) $xy = 3 \mod 6$
(D) $xy = 6 \mod 7$
(E) $x = 5 \mod 42$

Poll

$x = 5 \mod 7$ **and** $x = 5 \mod 6$
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What's true?

(A) $x + y = 2 \mod 7$
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(E) $x = 5 \mod 42$
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Poll

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(D) $xy = 6 \mod 7$
(E) $x = 5 \mod 42$
(F) $y = 39 \mod 42$

All true.

Xor

Computer Science:

Xor

Computer Science:
1 - True
0 - False

Xor

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$1 \vee 1 = 1$

Xor

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$A \oplus B$ - Exclusive or.

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Note: Also modular addition modulo 2!

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Property: $A \oplus B \oplus B = A$.

Xor

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Cryptography ...

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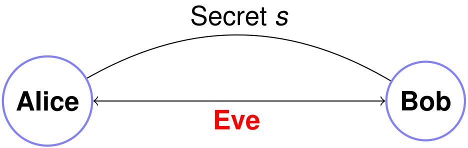
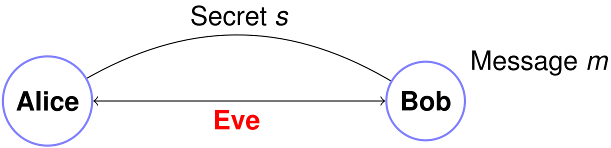


Cryptography ...



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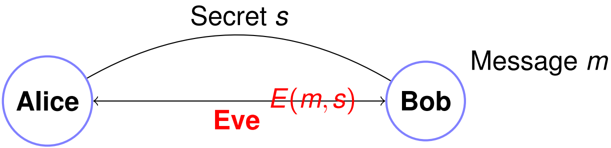

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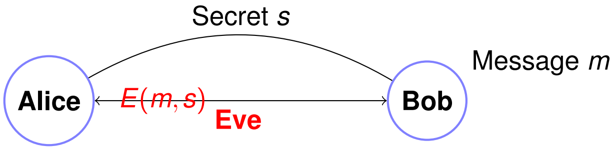

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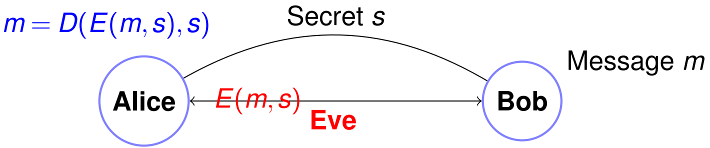

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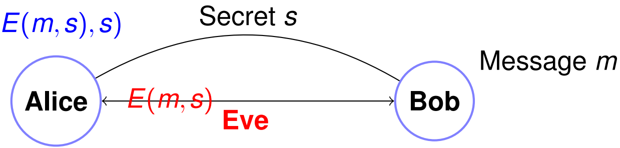
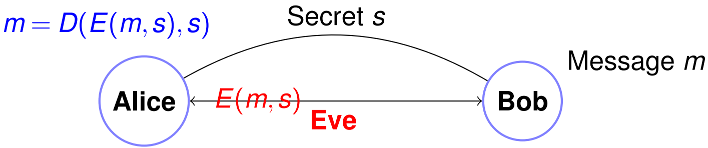


Example:

Cryptography ...



Example:
One-time Pad: secret s is string of length $|m|$.

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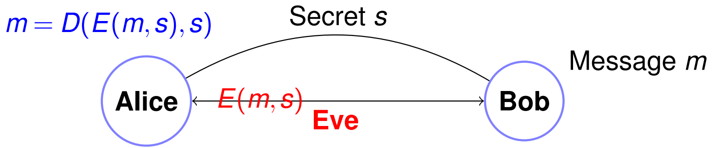

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$m = 1010101110101101$

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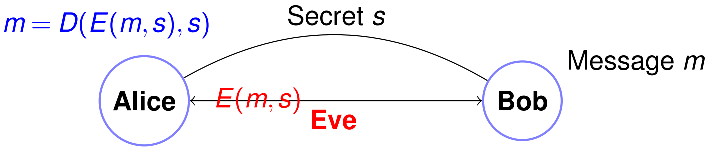

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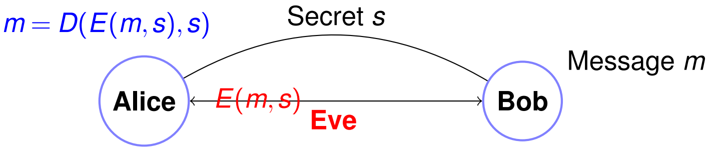

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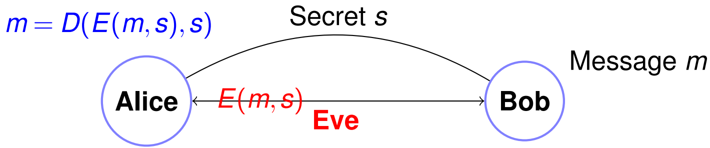

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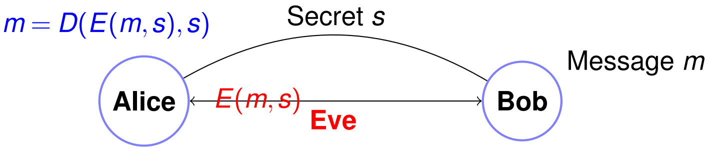

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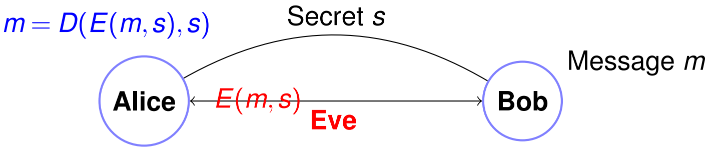

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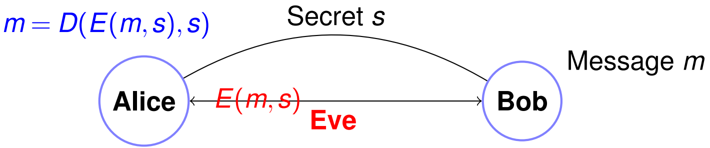

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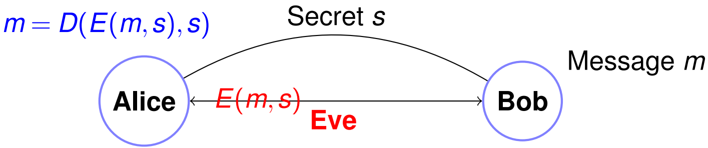

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Disadvantages:

Cryptography ...

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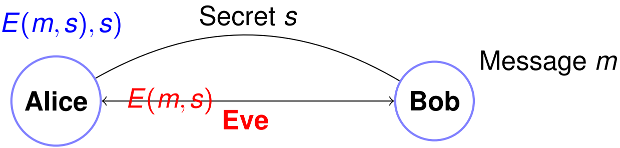

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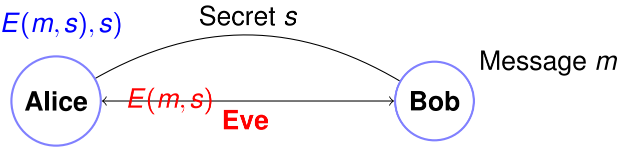

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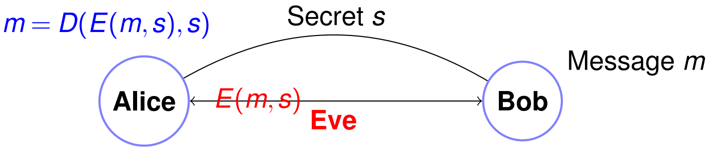

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Public key crypography.

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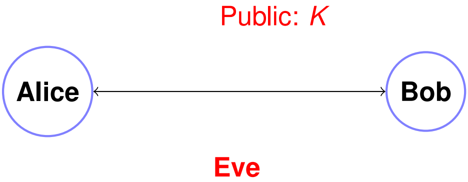

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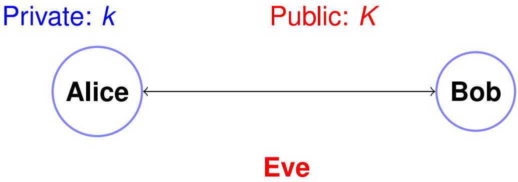

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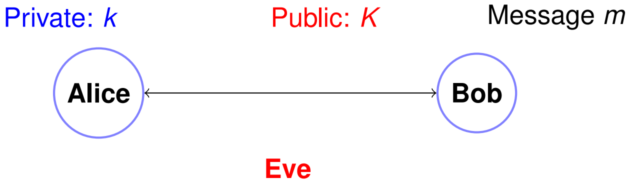

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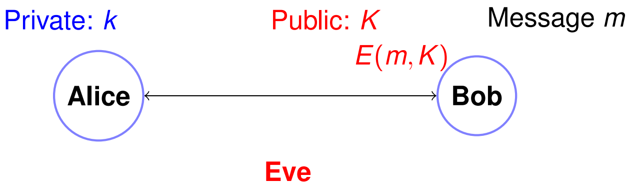

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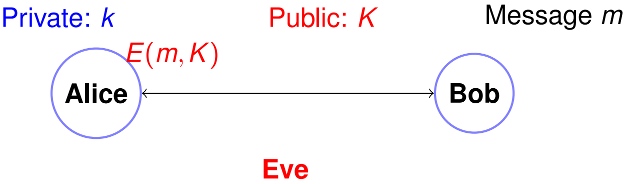

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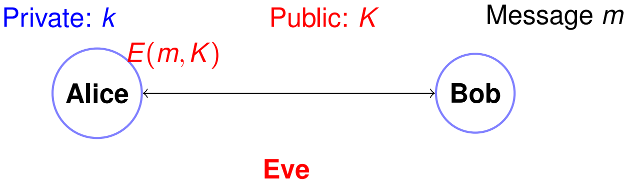

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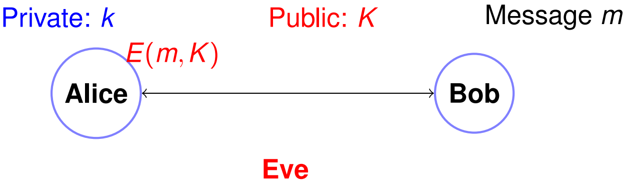

Everyone knows key K!

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Bob (and Eve

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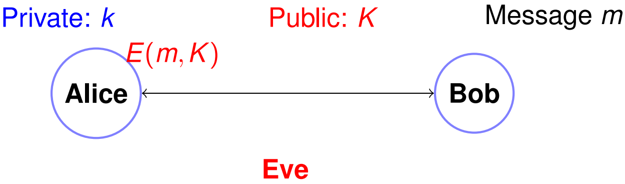

Everyone knows key K!
Bob (and Eve and me

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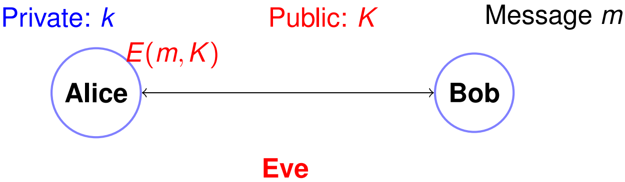

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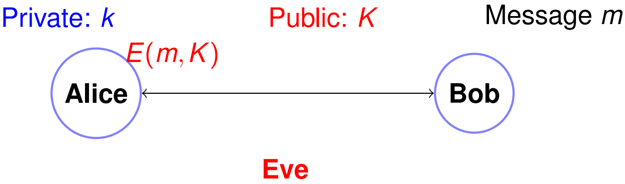

Everyone knows key K!
Bob (and Eve and me and you and you ...) can encode.

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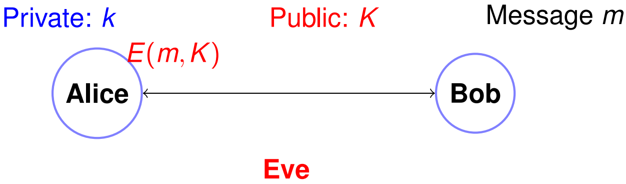

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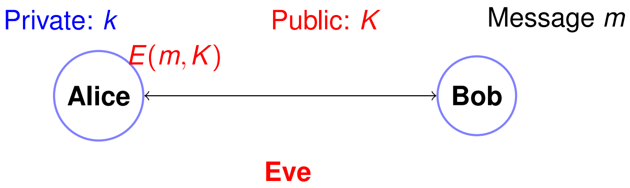

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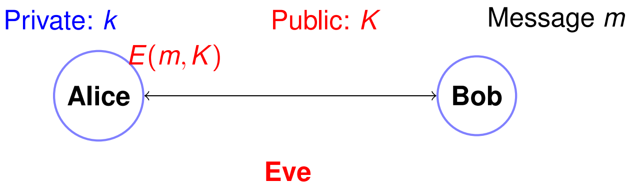

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Is this even possible?

Is public key crypto possible?

[1] Typically small, say $e = 3$.

Is public key crypto possible?

No. In a sense.

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Encode every message: $E(m', K)$. Check if Bob's: $E(m, K)$.

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Too slow. Does it have to be slow?

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Let $N = pq$.

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Choose e relatively prime to $(p-1)(q-1)$.[1]

Compute $d = e^{-1} \mod (p-1)(q-1)$.

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Pick two large primes p and q.

Let $N = pq$.

Choose e relatively prime to $(p-1)(q-1)$.[1]

Compute $d = e^{-1} \mod (p-1)(q-1)$.
Announce $N (= p \cdot q)$ and e: $K = (N, e)$ is my public key!

[1] Typically small, say $e = 3$.

Is public key crypto possible?

No. In a sense.
Encode every message: $E(m', K)$. Check if Bob's: $E(m, K)$.
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Iterative Extended GCD.

Example: $p = 7$, $q = 11$.

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Confirm:

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$d = e^{-1} = -17 = 43 = \pmod{60}$

Encryption/Decryption Techniques.

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Public Key: $(77, 7)$

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$E(2) = 2^e = 2^7 \equiv 128$

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$E(2) = 2^e = 2^7 \equiv 128 = 51 \pmod{77}$

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Obvious way: 43 multiplications.

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In general, $O(N)$ or $O(2^n)$ multiplications!

Repeated squaring.

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5 more multiplications.

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Repeated Squaring took 8 multiplications versus 42.

Repeated Squaring: x^y

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Repeated squaring $O(\log y)$ multiplications versus y!!!

1. x^y: Compute x^1,

Repeated Squaring: x^y

Repeated squaring $O(\log y)$ multiplications versus y!!!

1. x^y: Compute $x^1, x^2,$

Repeated Squaring: x^y

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Base case: $x^0 = 1$.

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Modular Exponentiation: $x^y \mod N$.

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Similar, not same, but useful.

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Each of $2, \ldots (p-1)$ has an inverse modulo p, solve to get...

$$a^{(p-1)} \equiv 1 \mod p.$$

□

Poll

Mark what is true.

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(A) $2^7 = 1 \mod 7$
(B) $2^6 = 1 \mod 7$
(C) $2^1, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7$ are distinct mod 7.
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$$y = x \pmod{p} \text{ and } y = x \pmod{q} \implies y = x \pmod{pq}.$$

RSA decodes correctly..

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All steps are polynomial in $O(\log N)$, the number of bits.

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Breaking in general sense $\implies$ factoring algorithm.

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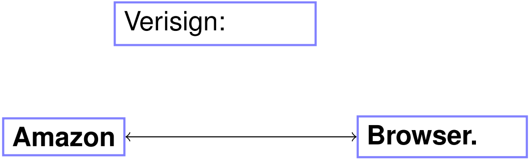
Signature can be verified by browser, since it has key.

Signatures using RSA.



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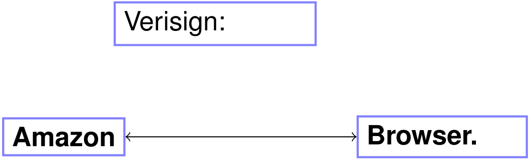

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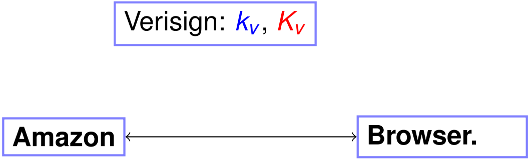

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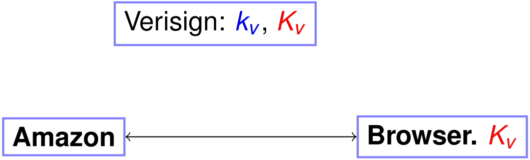

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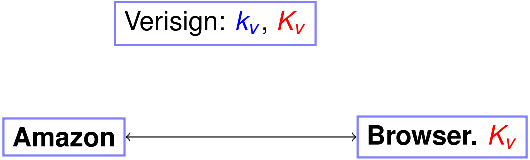

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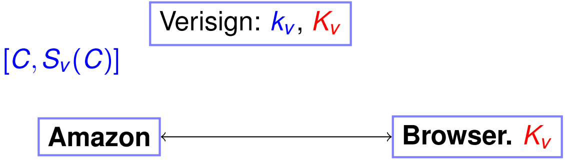

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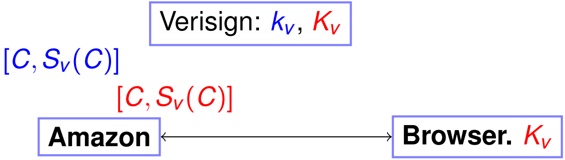

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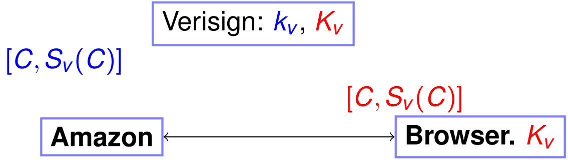

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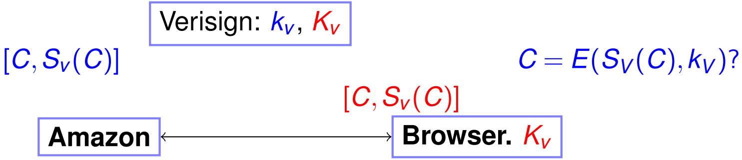

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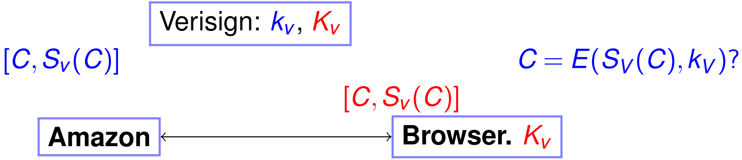

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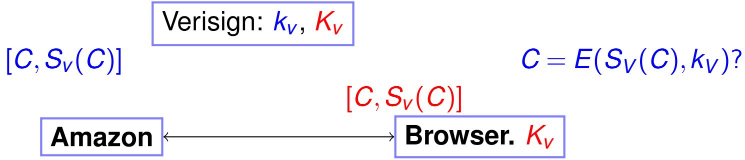

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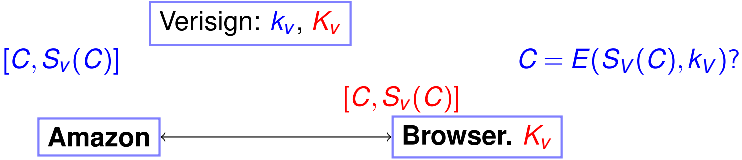

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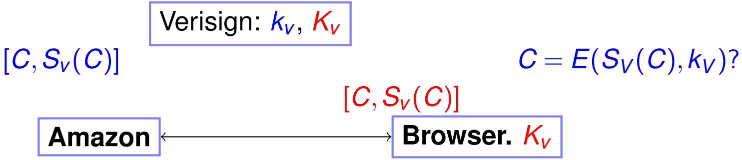

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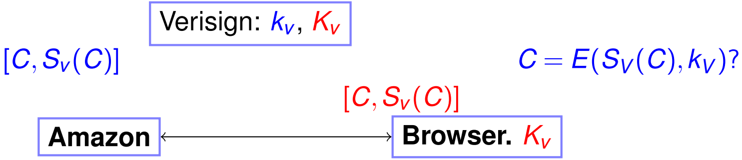

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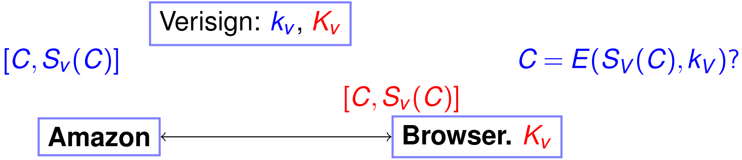

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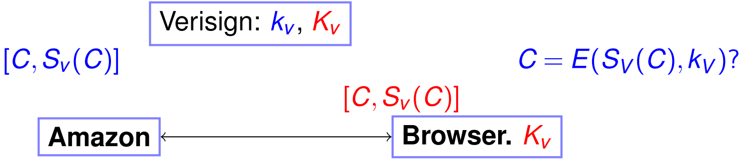

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Security: Eve can't forge unless she "breaks" RSA scheme.

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(B) and (D).

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